Connecting Fractions, Decimals and Mixed Numbers

(Using Spaced Repetition)

Fractions and Decimals Connected Via Fractional Number Lines Solidifying Student Understanding

Grades 3/4 - 9

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January, 2018

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To state the obvious, students' complete understanding of the decimal – fraction connection from both a conceptual and pragmatic computational proficiency is vitally important in arithmetic, algebra, trigonometry and geometry. Both the Texas Essential Knowledge or Skills (TEKS) and Common Core State Standards (CCSS) fourth and fifth grades math standards stipulate and accentuate student understanding of decimals and fractions in different formats. Both standards specify the importance of mastering both the computational and conceptual meaning of these mathematical elements. Consequently, elementary teachers present math lesson after math lesson attempting to ensure their students' comprehensive understanding of fractions and decimals.

Despite this elementary fraction and decimal preparatory work, middle and high school students often demonstrate a lack of fluidity when mathematically transitioning between equivalent forms of fractions and decimals. This lack of numeracy dramatically affects their problem solving ability and mathematical understanding of algebra and geometry. One of the main reasons that students do not master proper fractions, improper fractions, mixed numbers and decimals is that these topics are invariably taught discretely, but many teachers do not transition their lessons to a more global understanding between these mathematical elements. Hence, in doing so, these four interrelated skills 'float' in students' mathematics world and appear unconnected to one another. For example, many students may be able to visualize a pictorial model of an improper fraction such as 7/5, but they are not able to comfortable and mentally transform that fraction to an equivalent decimal of 1.4 or a mixed number of 1 2/5. Students often do not realize that these three quantities are not only equal, but all three elements may be represented by a similar pictorial model that is pedagogically introduced at the end of third grade or during their fourth and fifth grade elementary school years.

This short pedagogical document is intended to present a detailed and specific pedagogical methodology that connects fractional and decimal quantities both conceptually and computationally. This process is efficiently accomplished through an instructional methodology called "Spaced Repetition." The interested educator is provided further information on this topic via the download of a free white paper at the website address in the footer of this document. There are also two spaced repetition blogs on that same website that provide basic information on this instructional method as well. This instructional process may appear complicated, but it is not. It is only lengthy to convey the process in written form. However, heightened student outcomes justify the time investment of reading this paper by any third grade through eighth grade mathematics teacher. Finally, the Appendix in this document contains all the fractional/decimal number lines that will afford a classroom teacher little preparation time in lesson planning and implementation of this pedagogy.

There are several dependent numeracy skills that students must demonstrate proficiency prior to the instructional implementation of this methodology. Those skills include the following: All four (4) single digit math facts (addition, subtraction, multiplication and division), converting an improper fraction to a mixed number and vice versa, and converting proper fractions to equivalent decimals. It is also beneficial that 5^{th} through 8^{th} grade students are proficient with the standard use of a bar above a repeating, non-terminating decimal (i.e. $0.\overline{33}$ or $0.1\overline{66}$).

Part 1: Fractional Number Line Parsing Determination

A majority of student are not accustomed to visualizing fractional number lines as an adjacent series of 'fraction bars' laid end-to-end. Hence, many students incorrectly count the 'lines' and not the 'spaces' between ***any*** two whole numbers on a fractional number line. A pedagogical means to prevent this all too common student mistake is by verbally stating to students, *"Count the equal spaces between any two whole numbers."* For example, in the fractional number line shown in Figure 1, students should count the equal spaces between *any* two whole numbers on the fractional number line. Initially, many students will think the number line is in halves or 2's. Those students <u>incorrectly</u> count the lines between *any* two whole numbers. However, if students count the equal spaces, they realize the fractional number line is parsed in 'thirds' – to infinity in either the positive or negative direction on the number line.

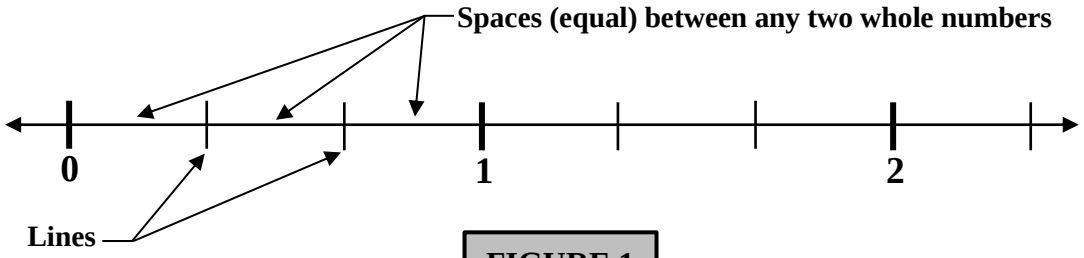


FIGURE 1

An effective visual to convey this meaning is by drawing/overlaying a series of 'fraction bars' directly on the fractional number line. This pictorial representation shown in Figure 2 significantly assists student understanding of a fractional number line with regard to his or her previous fraction bar classroom work in earlier lessons or grades.

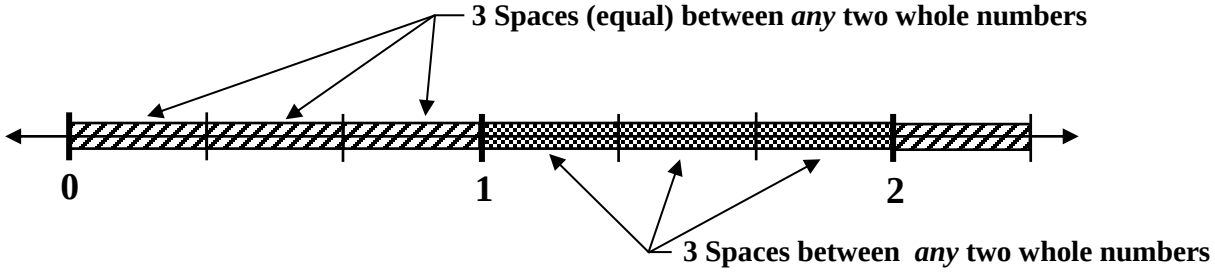


FIGURE 2

Part 1 Instruction: For ALL 3rd – 8th Grade Students

The first instructional step in this process is to ensure that all students can determine the correct parsing of a fractional number line – as outlined in the instructional procedure on page 3. It is recommended the procedure below be completed each day – repeatedly via spaced repetition instruction for 3 to 5 minutes until all students demonstrate mastery. In the Appendix, Blank Fractional Number Lines – Versions 1 through 4 are designed to accomplish this objective. There are four different versions of this fractional number line so students will not memorize the answers without comprehending the

concept. Finally, this procedure is the first stage in the instructional process for third through eighth grade students. It ensures all students begin Part 2 of the fraction – decimal learning process with the correct understanding of fractional number lines.

Correctly Determining the Parsing of a Fractional Number Line - Pedagogy

1.) **The teacher should print a classroom set plus 6 extra Xerox copies (student replacement) of Versions 1 through 4 of the blank fractional number lines located in the Appendix.**

2.) **Using a document camera or overhead, the teacher should select one of the four Versions and lay it on the overhead/document camera.**

3.) **The teacher should ask students, "On the fractional Number Line A (or B or C or D), what is the number of equal spaces equally divided/parsed between any two whole numbers?"**

4.) **Students usually respond incorrectly, at first. Note: They usually count the lines, not the spaces. The teacher should say, "*We must count the SPACES between any two whole numbers.*" On the number line, the teacher can make check marks for each space as he or she counts them aloud. For example, in Figure 1 above, the teacher would make three (3) check marks between 0 and 1 OR 1 and 2 indicating the number line is equally parsed into thirds.**

5.) **The teacher should select another fractional number line and repeat the procedure, asking students the parsing of the fractional number line. It is recommend the teacher ask students to hold up their hand(s) with the correct number of fingers indicating the spaces. In doing so, it prevents a student from calling out the answer. Hence, each student must do their own thinking. The teacher can also ask all student to only look forward and ask the back row for a show of raised hands with the correct number of fingers extended. Then, the next row in front of them, and so forth.**

6.) **After each number line, the teacher should make a small check for each equal space on the number line to demonstrate the correct answer – always counting the spaces between any two whole numbers on the fractional number line to determine the correct parsing of the number line. That specific fractional number line will always be parsed in that fractional element between any two whole numbers in either the positive or negative direction (e.g. if a particular number line is parsed in fourths, it is always in fourths).**

7.) **The teacher should continue this exercise each day using 'spaced repetition' instruction until ALL students demonstrate proficiency. The teacher should closely monitor students during these mini-lessons who are struggling to understand or have demonstrated prior academic difficulty to ensure he or she master the presented material.**

8.) **Finally, a summative assessment to test students is required and is easily completed using number line Version 1, 2, 3 or 4 – then, correct any student misunderstanding.**

9.) **Normal Duration: 2 to 4 minutes per day for 3 to 4 instructional days. Adjust as needed.**

Part 2: Fractional Number Line Labeling - Basic

With any given task that is successfully completed in any human endeavor, consistency of applied practice will be a key component. The effectiveness of a spaced repetition (instructional) system is absolutely dependent on the teacher's organizational ability and consistent daily practice with his or her students. The instructional procedures listed below should be implemented in short durations and repeated day-in and day-out until the desired student outcome is realized. The academic benefit of mastery of this specific mathematical content will provided a solid foundation and afford students success into their middle and high school mathematics courses.

The TEKS and Common Core math standards stipulate differing levels of fraction – decimal understanding. Hence, the instructional procedures below are separated by grade level or a range of grade level. Versions 5 through 10 "Blank Number Line – Fraction/Decimal" Student Practice Sheets can be located in the Appendix. A general solution to Version 5 through 10 is also available in the appendix for these practice sheets.

Part 2 Instruction: For 3rd – 4th Grade Students

Third and fourth grade students are developmentally ready to master the first series of fractional number lines to mastery for mixed numbers, proper fractions and improper fractions. Second semester third grade students and many fourth graders are not taught an equivalent conversion between mixed numbers and improper fractions and vice versa. Hence, a teacher may convey to his or her students to infer a mixed number as whole number plus a proper fraction as a logical combination when completing fractional number lines. For example, with minimal practice, students will have little to no difficulty writing the proper and improper fractions for any given fractional number line. Since the whole numbers are also shown on the number line, students easily recognize that a division past any whole number location is the whole number plus a proper fraction. Hence, if the fractional number line is parsed in fourths, the first line after any whole number on the number line must be the whole number (i.e. 1, 2, 3, etc.) plus the proper fraction 1/4. This basic understanding is solidified when a pictorial model is drawn and the equivalency between an improper fraction and a mixed number is the exactly the same drawn model or diagram. Finally, there is not an academic need to illustrate equivalent decimals in these grade levels unless the teacher desires to do so. If the teacher elects to instruct equivalent decimals to students at this age, it is strongly recommended that halves and fourths be predominately used to relate the fractions/decimals to money – a math area in which it is highly likely students have had prior exposure.

The teacher should stress that <u>every</u> fraction may always be represented as one of two physical mathematical expressions. First, a fraction is an equal parts of a group. Second, <u>all</u> fractions are division problems. Hence, the fraction 1/4 shown in Figure 3 may be represented as either <u>part of a group</u> (e.g. 1 equal part of 4 shaded) or a <u>division problem</u> (e.g. 1/4 equals 0.25 via a division problem).

The proper fraction 1/4 mathematically means:

A.) Part of a group: 

B.) A division problem:

$$1 \div 4 = 4\overline{)1.00}$$

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{8\downarrow} \\ 20 \\ \underline{20} \end{array}$$

FIGURE 3

3rd and 4th Grade Student Labeling of a Fractional Number Line - Pedagogy

1.) **The teacher should print a classroom set plus 6 extra Xerox copies (student replacement) of Versions 5 through 10 of the blank fractional number lines located in the Appendix.**

2.) **Using a document camera or overhead, the teacher should select one of the six (6) number line versions and lay it on the overhead/document camera.**

3.) **The teacher must model the exercise in a direct teach modus operandi, verbally communicating her work so students are able to visually see the teacher's work and hear his or her thinking. The teacher should begin by determining the divisional parsing of the fractional number line as was done in Part 1 above. It is recommended the teacher use the phrase, *"Count the spaces and label the lines."* Note: The teacher should secure (all) students' undivided attention prior to beginning the direct teach modelling.**

4.) **After the teacher has completed the modelling process, the spaced repetition instruction work should be *actively* completed quickly each day – 4 to 10 minutes total time. Hence, only one or two of the number lines (i.e. A, B, C or D) provided on each version should be completed each day in that time period. The teacher should have a blank number line version Xeroxed for each student that they reuse each day until all number lines have been completed. It is also recommended that the teacher Xerox a few extra copies in the event that a child loses their number line from the previous day.**

5.) **On the first days of implementation of this process, it is highly recommended to proceed via a slow and steady instructional rate to ensure that all students complete their work with understanding. After selecting a number line version (i.e. 5 through 10) and all students possess the same Xeroxed copy at his or her desk, the teacher selects a number line (i.e. A, B, C or D). Both teacher and students should complete the work together during the guided practice. Determination of the number line's fractional parsing is the first step.**

6.) **The teacher should guide the students through the process without allowing students to get ahead of him/her. The teacher should control the learning and vocabulary process by allowing students to only complete the proper fractions and stop. After the proper fractions are complete, then the students continue and complete the improper fractions. (See Figure 4 for a completed example.) The teacher should ask his or her students, "How do you know your number line is correctly labeled?" The teacher should verify the improper fraction accuracy by dividing the numerator by the denominator to ensure the quotient equals the whole number on the number line. For example, 4/4 equals 4 ÷ 4 = 1 whole, and 8/4 equals 8 ÷ 4 = 2, if the number line is in fourths, as is shown in Figure 4.**

7.) **After students finish the proper and improper fraction number lines, the teacher should require students to draw a picture of a proper fraction and an improper fraction. An example of that work is shown on the completed number line in Figure 5.**

8.) **After 4 to 6 class days of repetitive work with proper and improper fractions, the teacher should introduce mixed numbers on the number line as shown on Figure 6.**

3rd and 4th Grade Student Labeling of a Fractional Number Line - Continued

9.) Note: It is important when working with mixed numbers that students write the mixed number that is located at each whole number location. For example, if the number line is in fourths, the mixed number at '1' equals 1 0/4 and at '2,' equals 2 0/4. These mixed numbers are used in Part 3 of the Fraction-Decimal Connection methodology.

10.) As students complete (daily) the mixed numbers and both proper and improper fractions on the fractional number lines to mastery levels, the teacher should (again) require students to draw a picture each day of both a proper fraction and mixed number/improper fraction. In doing so, students understand that at each location on the number line, the mixed number is equivalent to the improper fraction. Students should write the equivalency to ensure understanding, as shown in figure 7.

11.) A student summative assessment of this process can be completed using Versions 5 – 10. Any student misunderstandings should be promptly addressed. Normal duration: 4 to 10 minutes each day for 6 to 9 instructional days. Adjust as needed.

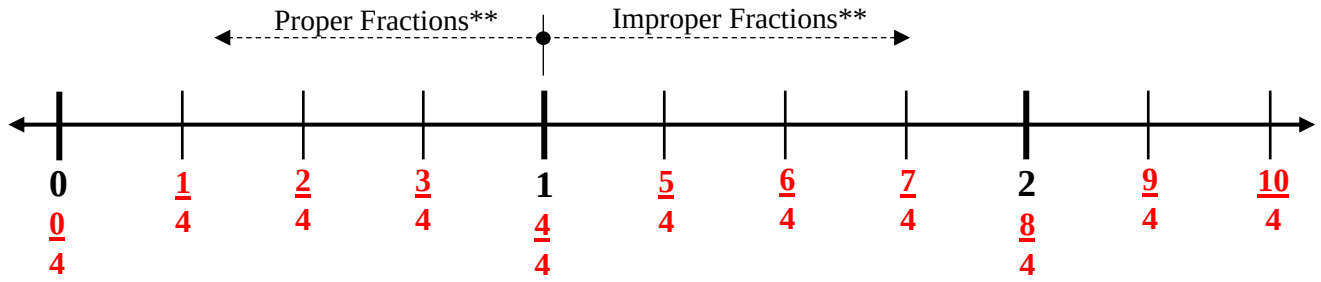


FIGURE 4

** Note to Teacher - FYI

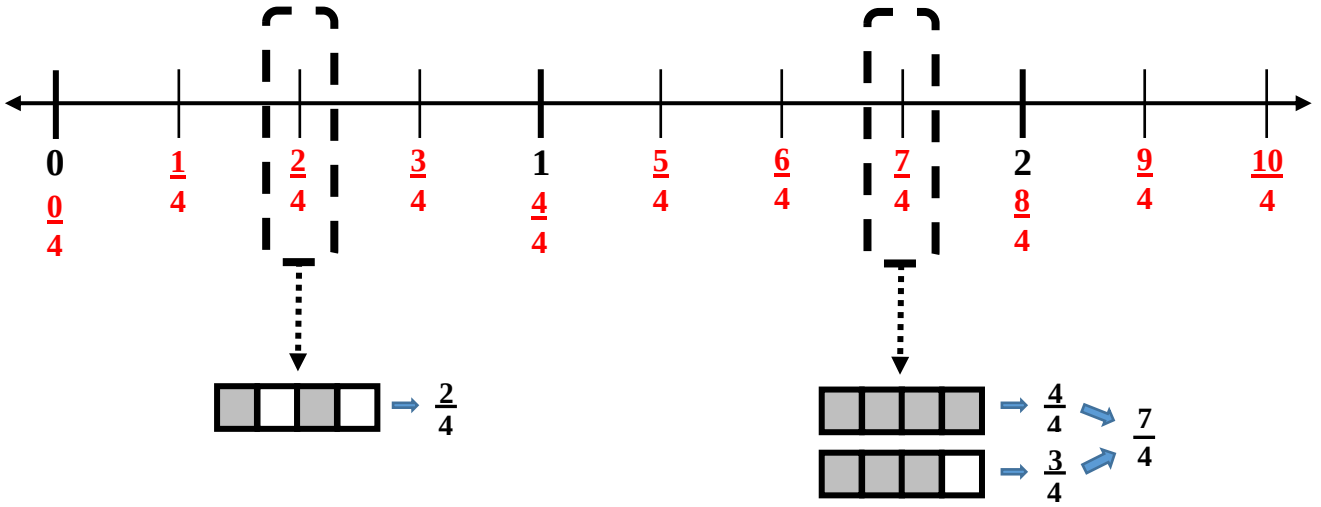


FIGURE 5

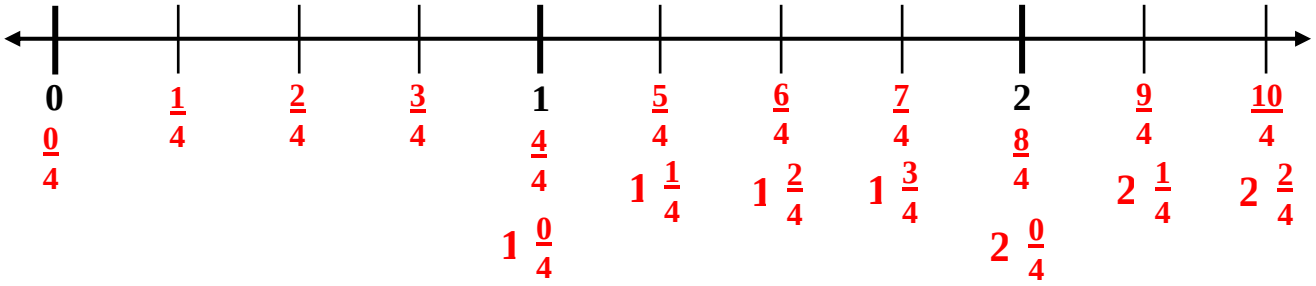


FIGURE 6

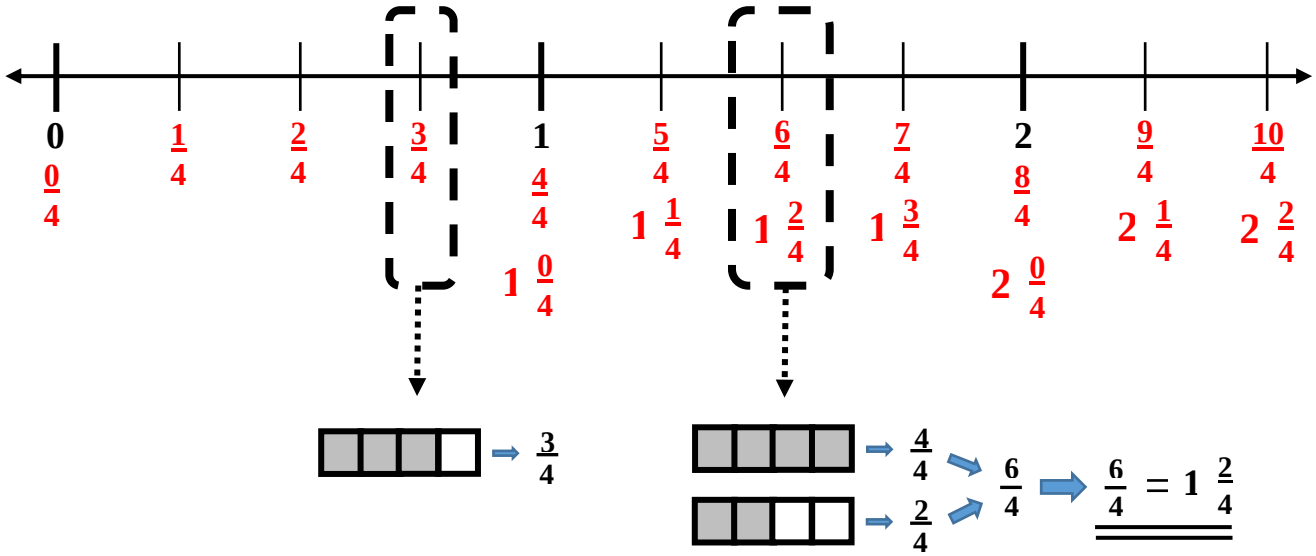


FIGURE 7

Part 2 Instruction: For 5th – 8th Grade Students

Fifth through eighth grade students are able to master the complete series of fractional number lines to mastery since they are required by the math standards to acquire decimal place value efficiency up to and including the thousandths place value. Math standards also require 5th grade level students to master equivalent conversions between mixed numbers and improper fractions as well as decimal equivalencies. Hence, a comprehensive understanding of the faction-decimal connection is possible.

In Figure 3, students were exposed to the decimal equivalencies between fraction and decimals via the division process. It is highly recommended that classroom teachers discretely practice this numeracy skill until students are proficient. Students should also understand the use of the bar over non-terminating, but repeating decimals. For example, for proper fractions '1/3' and '1/6,' students should be adept at dividing the fraction and computing decimal equivalencies – and written as $0.\overline{33}$ and $0.1\overline{66}$).

5th through 8th Grade Student Labeling of a Fractional Number Line - Pedagogy

1.) **The teacher should print Versions 5 through 10 of the number lines in the Appendix for classroom use. It is recommended to print several extra Xerox copies of each version in the event that a student loses their number line practice sheet from the day prior.**

2.) **The pedagogical steps and Figures presented in "*3rd and 4th Grade Student Labeling of a Fractional Number Line"* should be carefully read and reviewed. The procedures for mixed numbers, proper and improper fractions is exactly the same. 5th grade students and middle school students are continuing this work by completing the decimal portion of the fractional number line.**

3.) **Using a document camera or overhead, the teacher should select one of the six (6) number line versions (Version 5 through 10) and lay it on the overhead/document camera.**

4.) **As above, the teacher must model the exercise in a direct teach mode, verbally communicating his or her work so students are able to visually see the teacher's work and hear his or her thinking as decimals are added to the fractional number lines.**

5.) **Convert a proper fraction on the number line to an equivalent decimal via a simple division problem. Any proper fraction may be used (e.g. ½ = 0.5 and ¼ = 0.25, etc.) Note: A common student mistake is to not place the numerator 'under' the division 'house' and when the quotient is computed – it is greater than 1 whole, not less. In this case, the expression "*roll the fraction to the right*" may assist in avoiding this misstep.**

6.) **The student and teacher should complete the fractional number line together for the first couple of times during guided instruction to ensure student understanding: proper fractions, improper fractions, mixed numbers and decimals. Figure 8 provides a completed example of teacher/student work.**

7.) **After each number line is completed, the teacher should require students to draw a pictorial model of a proper fraction and its decimal equivalent – stating the equivalency with an equal sign. Finally, the student should also complete a similar pictorial model and state the equivalency for an improper fraction, mixed number and decimal. Figure 9 illustrates this process as a general solution.**

8.) **Give a summative assessment with a paper pencil Version 5 through 10, and correct any student misunderstandings. Normal duration: 7 to 10 minutes each day for an additional 5 to 7 instructional days with decimals added into the fractional number line process. Adjust as needed.**

Instructional Note: Students will begin to memorize proper faction - equivalent decimal values after only a few iterations of the spaced repetition instructional process. This situation is inevitable. However, if students understand that all fractions are division problems and each student is mathematically capable to convert proper and improper fractions to equivalent decimals, the memorization of equivalent fractions and decimals is acceptable. It is recommended that classroom teachers continue to provide his or her students with proper fractions (e.g. 1/7, 1/9, 1/12, etc.) and require them to use the division process to compute decimal equivalencies as well as improper fractions (e.g. 7/5, 9/4, etc.) and their decimal equivalencies.

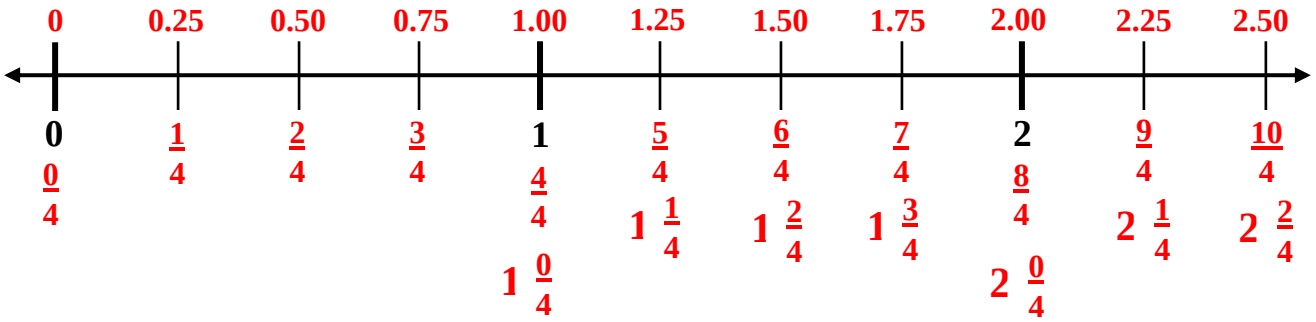
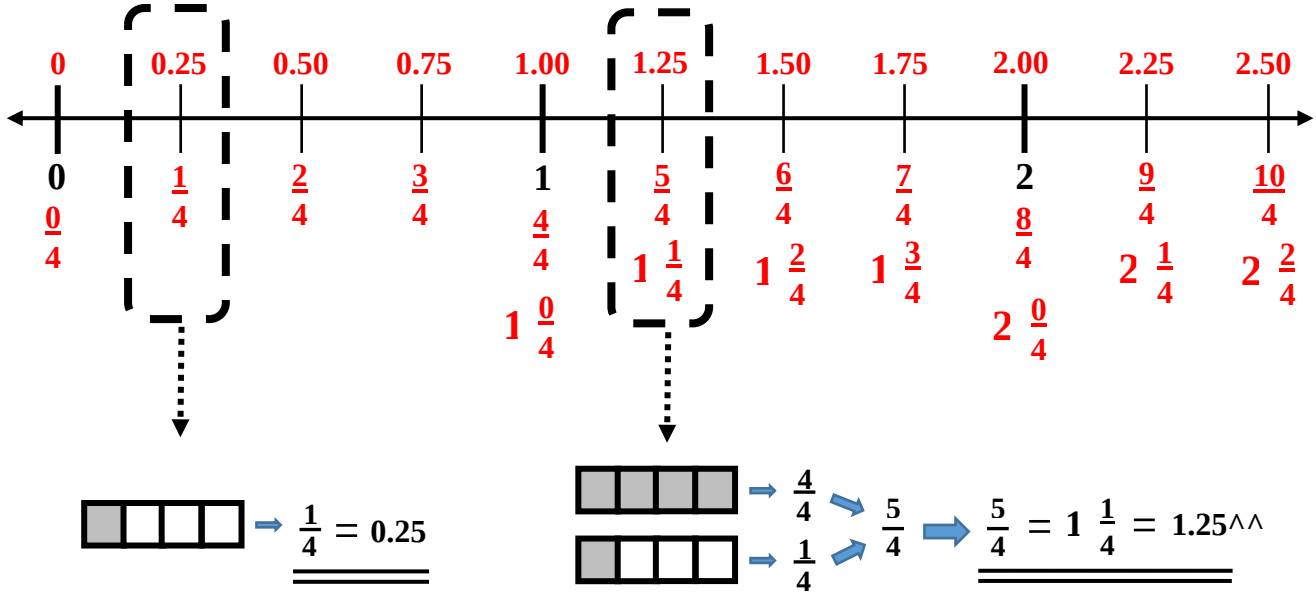


FIGURE 8



^^ Note to teacher: Students should reason that 1/4 = 0.25. Since, 4/4 = 1.00, then, 1.00 + 0.25 equals 1.25

FIGURE 9

Part 3: Fractional Number Line Labeling - Advanced

This last series of fractional number lines is designed for 5th grade through 8th grade students. This last set of factional number lines is relatively easy if students are mathematically capable to convert an improper fraction to an equivalent mixed number and they understand that any whole number can be rewritten as a whole number and a zero numerator proper fraction (e.g. the whole number '1' is equivalent to 1 0/4 or the whole number '2' is equivalent to 2 0/3). Finally, students must also be mathematically adept at converting simple proper fractions to equivalent decimals.

The spaced repetition instructional process is exactly the same as before with teacher modelling; however, the teacher should Xerox the Blank Number Lines – Version 11 and 12 for his or her students

from the Appendix. The teacher should begin with a whole number, as shown in Figure 10 with the whole number '3.' Equivalently, the first mixed number is 3 0/4. The student can complete the mixed numbers, and convert 3 0/4 to an equivalent improper fraction (e.g. 12/4). Since the number line is parsed in fourths, the proper fraction of 1/4 is equivalent to 0.25. Hence, the equivalent decimal of 13/4 or 3 1/4 is equal to 3.25. Each student can complete the fractional number line, accordingly. By the third day of practice, the students are usually incredibly proficient at the entire process.

Finally, the teacher should always choose a point on the number line and the students draw a pictorial of the decimal, mixed number and improper fraction equivalency. This process is illustrated in Figure 11. The teacher should use the completely blank number lines – Versions 13 through 16 (e.g. number lines without set whole numbers) from the Appendix and provide different whole numbers of their choosing to provide his or her students additional practice. This last process usually requires 7 to 10 minutes per day for approximately 5 to 7 days. The teacher may adjust as needed. A summative assessment should be proctored to ensure students have mastered the fractional-decimal connection using a Versions 13 through 16 using whole numbers of the teacher's choosing.

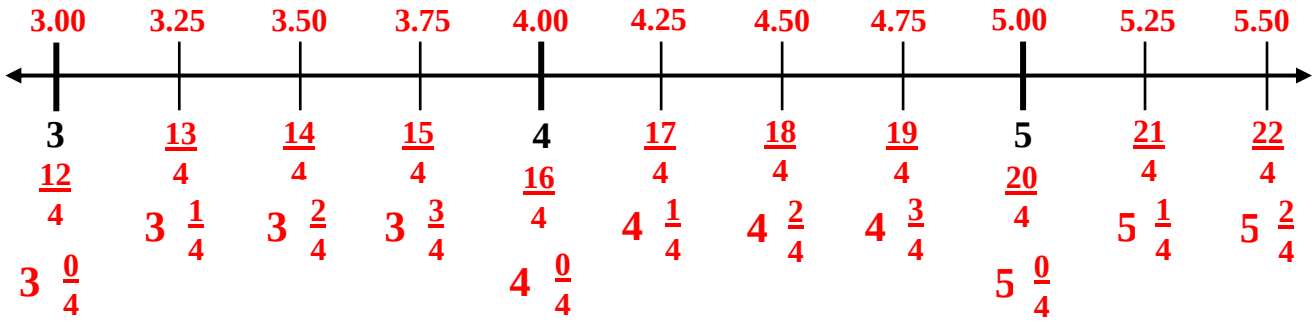


FIGURE 10

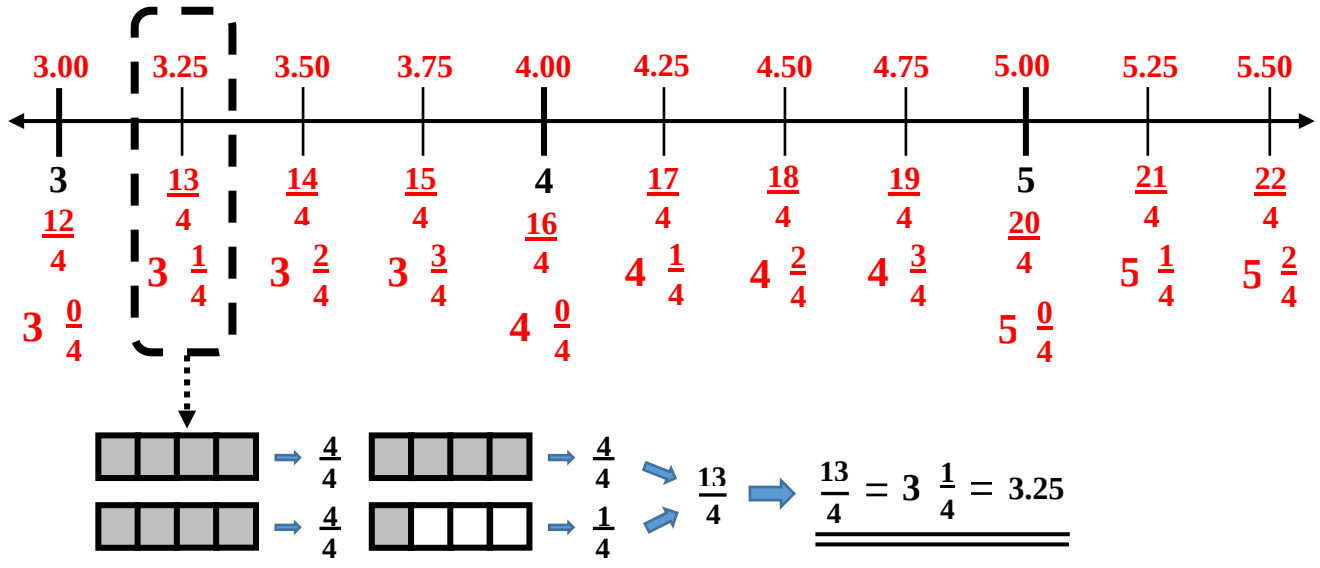


FIGURE 11

Connecting Fractions and Decimals Using Spaced Repetition

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Part 1: Fractional Number Line Parsing Determination

A majority of student are not accustomed to visualizing fractional number lines as an adjacent series of 'fraction bars' laid end-to-end. Hence, many students incorrectly count the 'lines' and not the 'spaces' between ***any*** two whole numbers on a fractional number line. A pedagogical means to prevent this all too common student mistake is by verbally stating to students, *"Count the equal spaces between any two whole numbers."* For example, in the fractional number line shown in Figure 1, students should count the equal spaces between *any* two whole numbers on the fractional number line. Initially, many students will think the number line is in halves or 2's. Those students <u>incorrectly</u> count the lines between *any* two whole numbers. However, if students count the equal spaces, they realize the fractional number line is parsed in 'thirds' – to infinity in either the positive or negative direction on the number line.

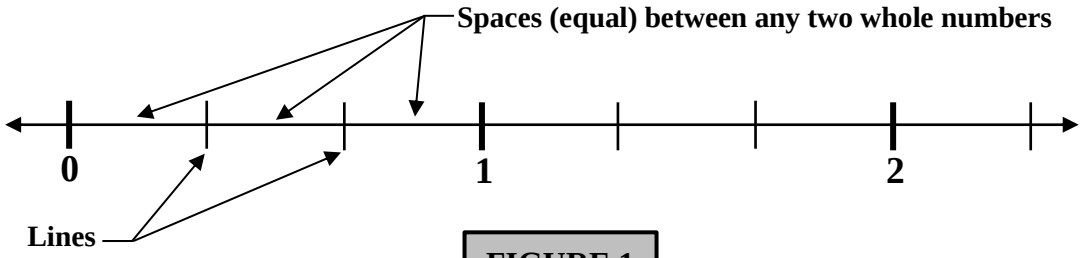


FIGURE 1

An effective visual to convey this meaning is by drawing/overlaying a series of 'fraction bars' directly on the fractional number line. This pictorial representation shown in Figure 2 significantly assists student understanding of a fractional number line with regard to his or her previous fraction bar classroom work in earlier lessons or grades.

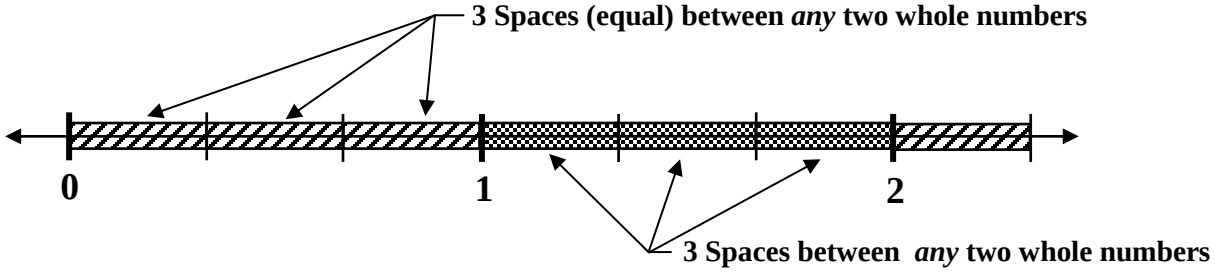


FIGURE 2

Part 1 Instruction: For ALL 3rd – 8th Grade Students

The first instructional step in this process is to ensure that all students can determine the correct parsing of a fractional number line – as outlined in the instructional procedure on page 3. It is recommended the procedure below be completed each day – repeatedly via spaced repetition instruction for 3 to 5 minutes until all students demonstrate mastery. In the Appendix, Blank Fractional Number Lines – Versions 1 through 4 are designed to accomplish this objective. There are four different versions of this fractional number line so students will not memorize the answers without comprehending the

concept. Finally, this procedure is the first stage in the instructional process for third through eighth grade students. It ensures all students begin Part 2 of the fraction – decimal learning process with the correct understanding of fractional number lines.

Correctly Determining the Parsing of a Fractional Number Line - Pedagogy

1.) **The teacher should print a classroom set plus 6 extra Xerox copies (student replacement) of Versions 1 through 4 of the blank fractional number lines located in the Appendix.**

2.) **Using a document camera or overhead, the teacher should select one of the four Versions and lay it on the overhead/document camera.**

3.) **The teacher should ask students, "On the fractional Number Line A (or B or C or D), what is the number of equal spaces equally divided/parsed between any two whole numbers?"**

4.) **Students usually respond incorrectly, at first. Note: They usually count the lines, not the spaces. The teacher should say, "*We must count the SPACES between any two whole numbers.*" On the number line, the teacher can make check marks for each space as he or she counts them aloud. For example, in Figure 1 above, the teacher would make three (3) check marks between 0 and 1 OR 1 and 2 indicating the number line is equally parsed into thirds.**

5.) **The teacher should select another fractional number line and repeat the procedure, asking students the parsing of the fractional number line. It is recommend the teacher ask students to hold up their hand(s) with the correct number of fingers indicating the spaces. In doing so, it prevents a student from calling out the answer. Hence, each student must do their own thinking. The teacher can also ask all student to only look forward and ask the back row for a show of raised hands with the correct number of fingers extended. Then, the next row in front of them, and so forth.**

6.) **After each number line, the teacher should make a small check for each equal space on the number line to demonstrate the correct answer – always counting the spaces between any two whole numbers on the fractional number line to determine the correct parsing of the number line. That specific fractional number line will always be parsed in that fractional element between any two whole numbers in either the positive or negative direction (e.g. if a particular number line is parsed in fourths, it is always in fourths).**

7.) **The teacher should continue this exercise each day using 'spaced repetition' instruction until ALL students demonstrate proficiency. The teacher should closely monitor students during these mini-lessons who are struggling to understand or have demonstrated prior academic difficulty to ensure he or she master the presented material.**

8.) **Finally, a summative assessment to test students is required and is easily completed using number line Version 1, 2, 3 or 4 – then, correct any student misunderstanding.**

9.) **Normal Duration: 2 to 4 minutes per day for 3 to 4 instructional days. Adjust as needed.**

Part 2: Fractional Number Line Labeling - Basic

With any given task that is successfully completed in any human endeavor, consistency of applied practice will be a key component. The effectiveness of a spaced repetition (instructional) system is absolutely dependent on the teacher's organizational ability and consistent daily practice with his or her students. The instructional procedures listed below should be implemented in short durations and repeated day-in and day-out until the desired student outcome is realized. The academic benefit of mastery of this specific mathematical content will provided a solid foundation and afford students success into their middle and high school mathematics courses.

The TEKS and Common Core math standards stipulate differing levels of fraction – decimal understanding. Hence, the instructional procedures below are separated by grade level or a range of grade level. Versions 5 through 10 "Blank Number Line – Fraction/Decimal" Student Practice Sheets can be located in the Appendix. A general solution to Version 5 through 10 is also available in the appendix for these practice sheets.

Part 2 Instruction: For 3rd – 4th Grade Students

Third and fourth grade students are developmentally ready to master the first series of fractional number lines to mastery for mixed numbers, proper fractions and improper fractions. Second semester third grade students and many fourth graders are not taught an equivalent conversion between mixed numbers and improper fractions and vice versa. Hence, a teacher may convey to his or her students to infer a mixed number as whole number plus a proper fraction as a logical combination when completing fractional number lines. For example, with minimal practice, students will have little to no difficulty writing the proper and improper fractions for any given fractional number line. Since the whole numbers are also shown on the number line, students easily recognize that a division past any whole number location is the whole number plus a proper fraction. Hence, if the fractional number line is parsed in fourths, the first line after any whole number on the number line must be the whole number (i.e. 1, 2, 3, etc.) plus the proper fraction 1/4. This basic understanding is solidified when a pictorial model is drawn and the equivalency between an improper fraction and a mixed number is the exactly the same drawn model or diagram. Finally, there is not an academic need to illustrate equivalent decimals in these grade levels unless the teacher desires to do so. If the teacher elects to instruct equivalent decimals to students at this age, it is strongly recommended that halves and fourths be predominately used to relate the fractions/decimals to money – a math area in which it is highly likely students have had prior exposure.

The teacher should stress that <u>every</u> fraction may always be represented as one of two physical mathematical expressions. First, a fraction is an equal parts of a group. Second, <u>all</u> fractions are division problems. Hence, the fraction 1/4 shown in Figure 3 may be represented as either <u>part of a group</u> (e.g. 1 equal part of 4 shaded) or a <u>division problem</u> (e.g. 1/4 equals 0.25 via a division problem).

The proper fraction 1/4 mathematically means:

A.) Part of a group: 

B.) A division problem:

$$1 \div 4 = 4\overline{)1.00}$$

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{8\downarrow} \\ 20 \\ \underline{20} \end{array}$$

FIGURE 3

3rd and 4th Grade Student Labeling of a Fractional Number Line - Pedagogy

1.) **The teacher should print a classroom set plus 6 extra Xerox copies (student replacement) of Versions 5 through 10 of the blank fractional number lines located in the Appendix.**

2.) **Using a document camera or overhead, the teacher should select one of the six (6) number line versions and lay it on the overhead/document camera.**

3.) **The teacher must model the exercise in a direct teach modus operandi, verbally communicating her work so students are able to visually see the teacher's work and hear his or her thinking. The teacher should begin by determining the divisional parsing of the fractional number line as was done in Part 1 above. It is recommended the teacher use the phrase, *"Count the spaces and label the lines."* Note: The teacher should secure (all) students' undivided attention prior to beginning the direct teach modelling.**

4.) **After the teacher has completed the modelling process, the spaced repetition instruction work should be *actively* completed quickly each day – 4 to 10 minutes total time. Hence, only one or two of the number lines (i.e. A, B, C or D) provided on each version should be completed each day in that time period. The teacher should have a blank number line version Xeroxed for each student that they reuse each day until all number lines have been completed. It is also recommended that the teacher Xerox a few extra copies in the event that a child loses their number line from the previous day.**

5.) **On the first days of implementation of this process, it is highly recommended to proceed via a slow and steady instructional rate to ensure that all students complete their work with understanding. After selecting a number line version (i.e. 5 through 10) and all students possess the same Xeroxed copy at his or her desk, the teacher selects a number line (i.e. A, B, C or D). Both teacher and students should complete the work together during the guided practice. Determination of the number line's fractional parsing is the first step.**

6.) **The teacher should guide the students through the process without allowing students to get ahead of him/her. The teacher should control the learning and vocabulary process by allowing students to only complete the proper fractions and stop. After the proper fractions are complete, then the students continue and complete the improper fractions. (See Figure 4 for a completed example.) The teacher should ask his or her students, "How do you know your number line is correctly labeled?" The teacher should verify the improper fraction accuracy by dividing the numerator by the denominator to ensure the quotient equals the whole number on the number line. For example, 4/4 equals 4 ÷ 4 = 1 whole, and 8/4 equals 8 ÷ 4 = 2, if the number line is in fourths, as is shown in Figure 4.**

7.) **After students finish the proper and improper fraction number lines, the teacher should require students to draw a picture of a proper fraction and an improper fraction. An example of that work is shown on the completed number line in Figure 5.**

8.) **After 4 to 6 class days of repetitive work with proper and improper fractions, the teacher should introduce mixed numbers on the number line as shown on Figure 6.**

3rd and 4th Grade Student Labeling of a Fractional Number Line - Continued

9.) Note: It is important when working with mixed numbers that students write the mixed number that is located at each whole number location. For example, if the number line is in fourths, the mixed number at '1' equals 1 0/4 and at '2,' equals 2 0/4. These mixed numbers are used in Part 3 of the Fraction-Decimal Connection methodology.

10.) As students complete (daily) the mixed numbers and both proper and improper fractions on the fractional number lines to mastery levels, the teacher should (again) require students to draw a picture each day of both a proper fraction and mixed number/improper fraction. In doing so, students understand that at each location on the number line, the mixed number is equivalent to the improper fraction. Students should write the equivalency to ensure understanding, as shown in figure 7.

11.) A student summative assessment of this process can be completed using Versions 5 – 10. Any student misunderstandings should be promptly addressed. Normal duration: 4 to 10 minutes each day for 6 to 9 instructional days. Adjust as needed.

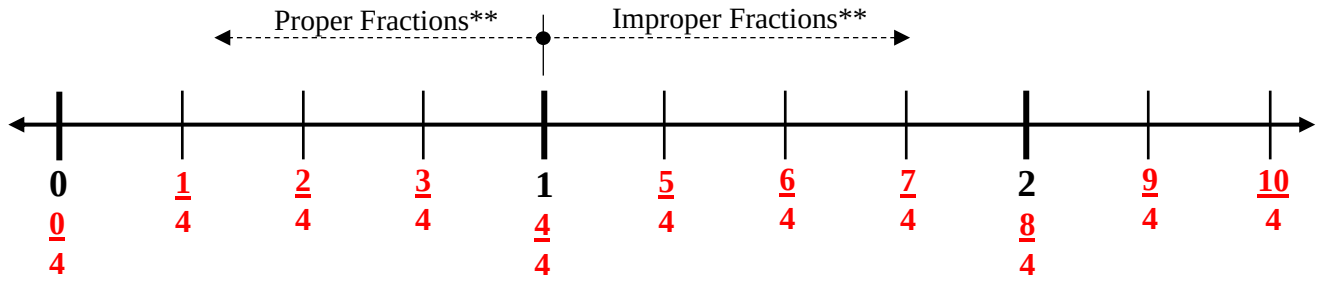


FIGURE 4

** Note to Teacher - FYI

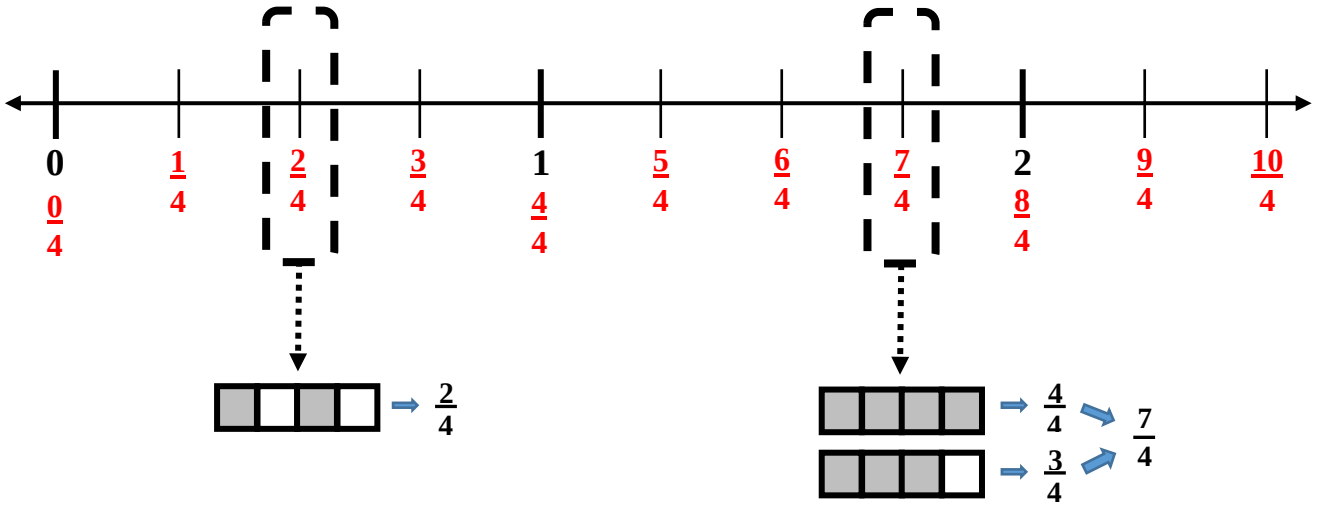


FIGURE 5

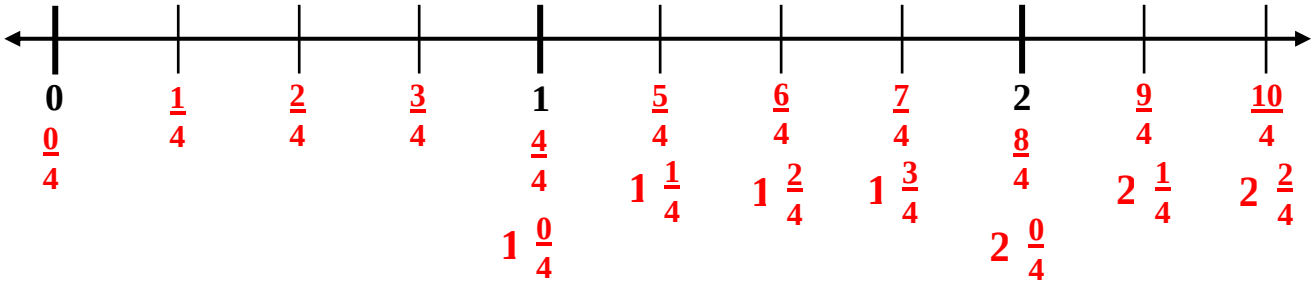


FIGURE 6

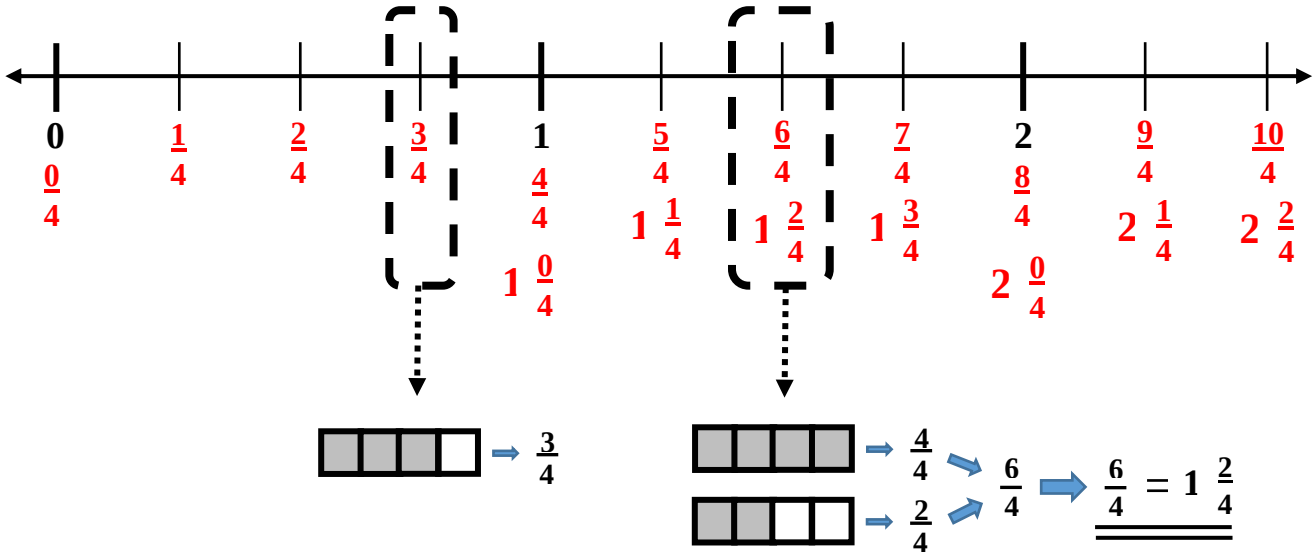


FIGURE 7

Part 2 Instruction: For 5th – 8th Grade Students

Fifth through eighth grade students are able to master the complete series of fractional number lines to mastery since they are required by the math standards to acquire decimal place value efficiency up to and including the thousandths place value. Math standards also require 5th grade level students to master equivalent conversions between mixed numbers and improper fractions as well as decimal equivalencies. Hence, a comprehensive understanding of the faction-decimal connection is possible.

In Figure 3, students were exposed to the decimal equivalencies between fraction and decimals via the division process. It is highly recommended that classroom teachers discretely practice this numeracy skill until students are proficient. Students should also understand the use of the bar over non-terminating, but repeating decimals. For example, for proper fractions '1/3' and '1/6,' students should be adept at dividing the fraction and computing decimal equivalencies – and written as $0.\overline{33}$ and $0.1\overline{66}$).

5th through 8th Grade Student Labeling of a Fractional Number Line - Pedagogy

1.) **The teacher should print Versions 5 through 10 of the number lines in the Appendix for classroom use. It is recommended to print several extra Xerox copies of each version in the event that a student loses their number line practice sheet from the day prior.**

2.) **The pedagogical steps and Figures presented in "*3rd and 4th Grade Student Labeling of a Fractional Number Line"* should be carefully read and reviewed. The procedures for mixed numbers, proper and improper fractions is exactly the same. 5th grade students and middle school students are continuing this work by completing the decimal portion of the fractional number line.**

3.) **Using a document camera or overhead, the teacher should select one of the six (6) number line versions (Version 5 through 10) and lay it on the overhead/document camera.**

4.) **As above, the teacher must model the exercise in a direct teach mode, verbally communicating his or her work so students are able to visually see the teacher's work and hear his or her thinking as decimals are added to the fractional number lines.**

5.) **Convert a proper fraction on the number line to an equivalent decimal via a simple division problem. Any proper fraction may be used (e.g. ½ = 0.5 and ¼ = 0.25, etc.) Note: A common student mistake is to not place the numerator 'under' the division 'house' and when the quotient is computed – it is greater than 1 whole, not less. In this case, the expression "*roll the fraction to the right*" may assist in avoiding this misstep.**

6.) **The student and teacher should complete the fractional number line together for the first couple of times during guided instruction to ensure student understanding: proper fractions, improper fractions, mixed numbers and decimals. Figure 8 provides a completed example of teacher/student work.**

7.) **After each number line is completed, the teacher should require students to draw a pictorial model of a proper fraction and its decimal equivalent – stating the equivalency with an equal sign. Finally, the student should also complete a similar pictorial model and state the equivalency for an improper fraction, mixed number and decimal. Figure 9 illustrates this process as a general solution.**

8.) **Give a summative assessment with a paper pencil Version 5 through 10, and correct any student misunderstandings. Normal duration: 7 to 10 minutes each day for an additional 5 to 7 instructional days with decimals added into the fractional number line process. Adjust as needed.**

Instructional Note: Students will begin to memorize proper faction - equivalent decimal values after only a few iterations of the spaced repetition instructional process. This situation is inevitable. However, if students understand that all fractions are division problems and each student is mathematically capable to convert proper and improper fractions to equivalent decimals, the memorization of equivalent fractions and decimals is acceptable. It is recommended that classroom teachers continue to provide his or her students with proper fractions (e.g. 1/7, 1/9, 1/12, etc.) and require them to use the division process to compute decimal equivalencies as well as improper fractions (e.g. 7/5, 9/4, etc.) and their decimal equivalencies.

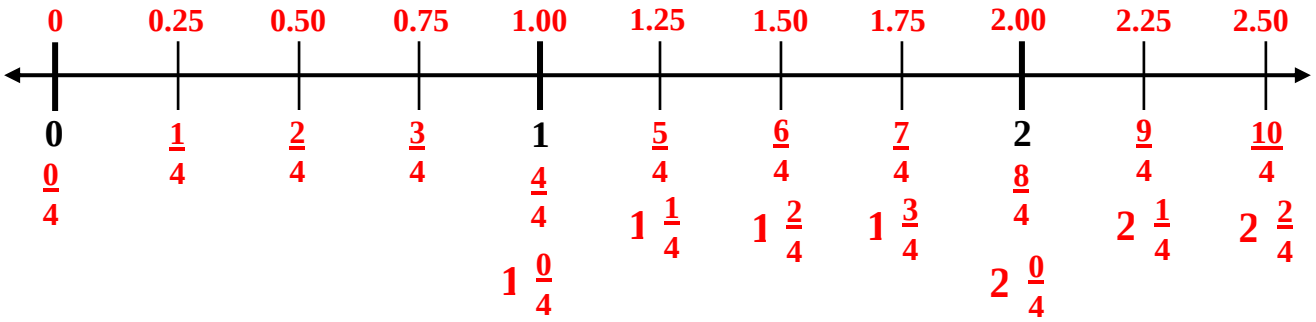
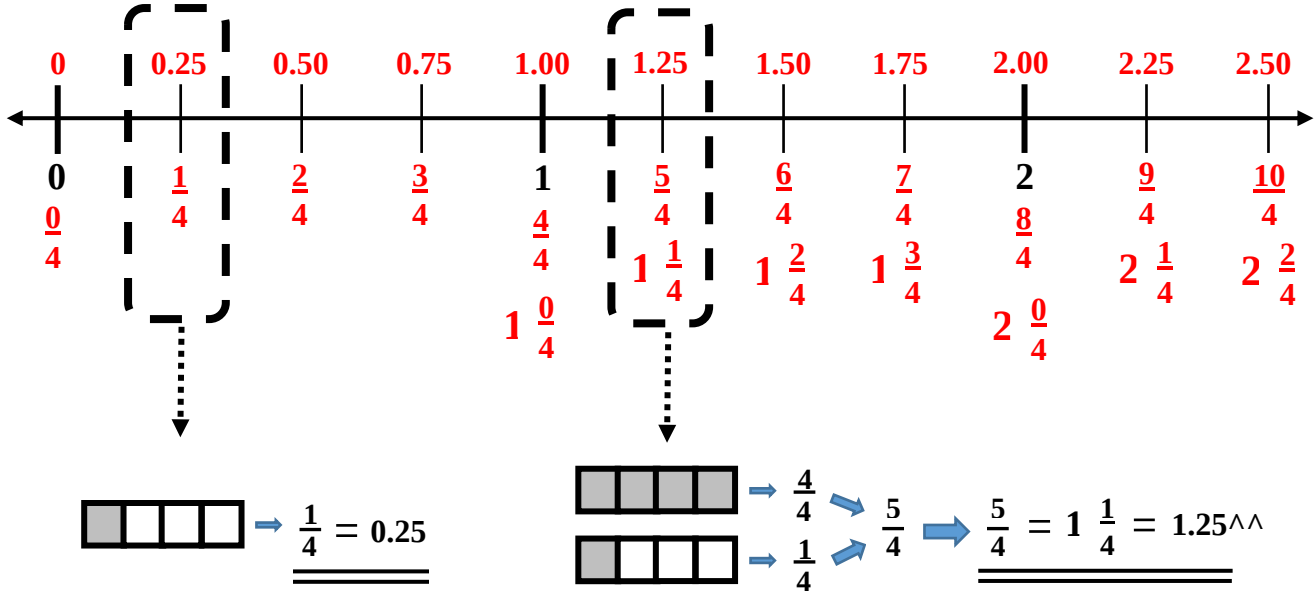


FIGURE 8



^^ Note to teacher: Students should reason that 1/4 = 0.25. Since, 4/4 = 1.00, then, 1.00 + 0.25 equals 1.25

FIGURE 9

Part 3: Fractional Number Line Labeling - Advanced

This last series of fractional number lines is designed for 5th grade through 8th grade students. This last set of factional number lines is relatively easy if students are mathematically capable to convert an improper fraction to an equivalent mixed number and they understand that any whole number can be rewritten as a whole number and a zero numerator proper fraction (e.g. the whole number '1' is equivalent to 1 0/4 or the whole number '2' is equivalent to 2 0/3). Finally, students must also be mathematically adept at converting simple proper fractions to equivalent decimals.

The spaced repetition instructional process is exactly the same as before with teacher modelling; however, the teacher should Xerox the Blank Number Lines – Version 11 and 12 for his or her students

from the Appendix. The teacher should begin with a whole number, as shown in Figure 10 with the whole number '3.' Equivalently, the first mixed number is 3 0/4. The student can complete the mixed numbers, and convert 3 0/4 to an equivalent improper fraction (e.g. 12/4). Since the number line is parsed in fourths, the proper fraction of 1/4 is equivalent to 0.25. Hence, the equivalent decimal of 13/4 or 3 1/4 is equal to 3.25. Each student can complete the fractional number line, accordingly. By the third day of practice, the students are usually incredibly proficient at the entire process.

Finally, the teacher should always choose a point on the number line and the students draw a pictorial of the decimal, mixed number and improper fraction equivalency. This process is illustrated in Figure 11. The teacher should use the completely blank number lines – Versions 13 through 16 (e.g. number lines without set whole numbers) from the Appendix and provide different whole numbers of their choosing to provide his or her students additional practice. This last process usually requires 7 to 10 minutes per day for approximately 5 to 7 days. The teacher may adjust as needed. A summative assessment should be proctored to ensure students have mastered the fractional-decimal connection using a Versions 13 through 16 using whole numbers of the teacher's choosing.

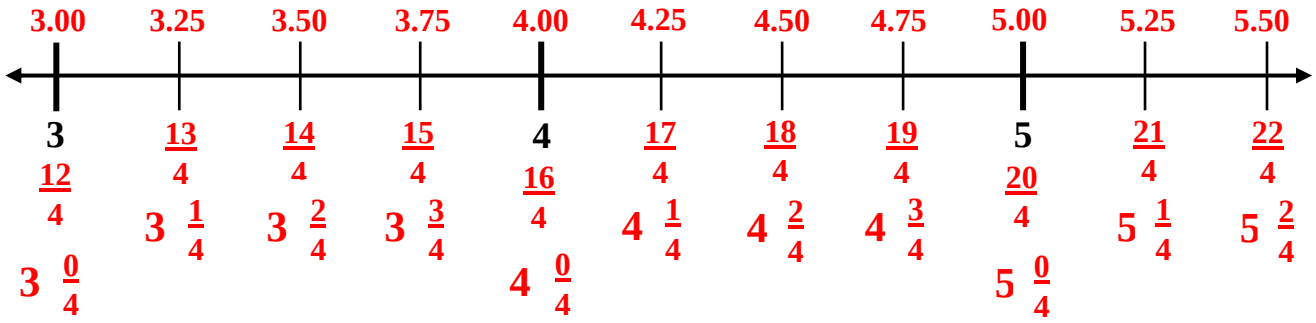


FIGURE 10

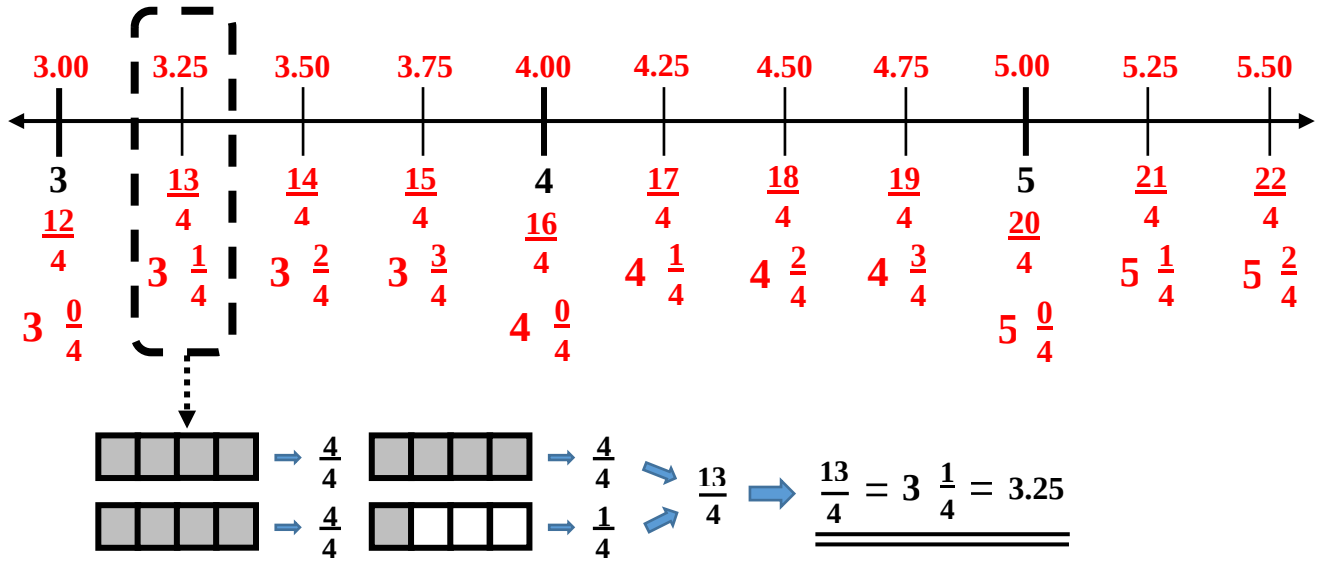


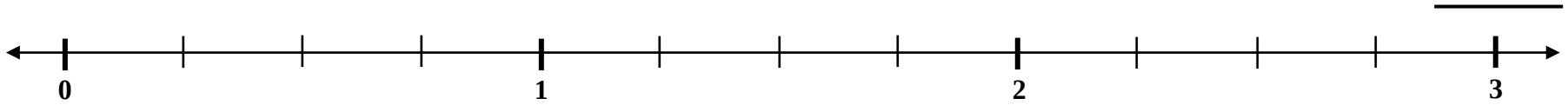
FIGURE 11

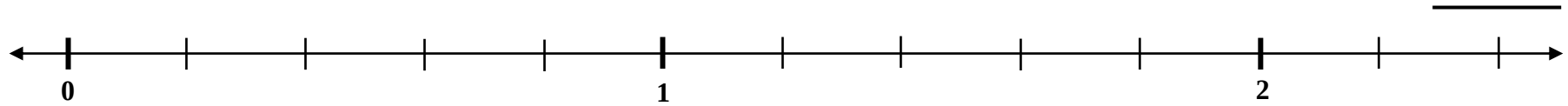
APPENDIX OF RESOURCES

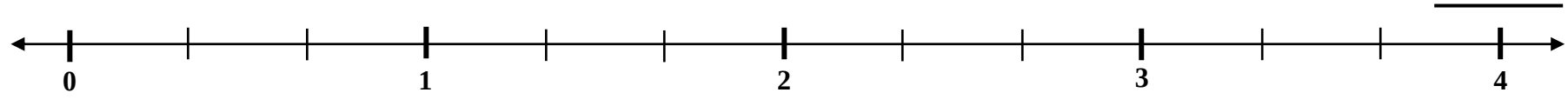
FRACTION/DECIMAL CONNECTION

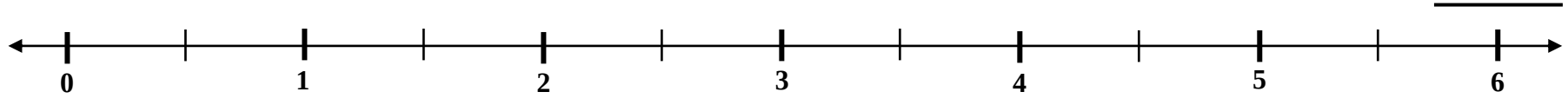
Blank Number Lines – Fraction/Decimal Connection – Version 1

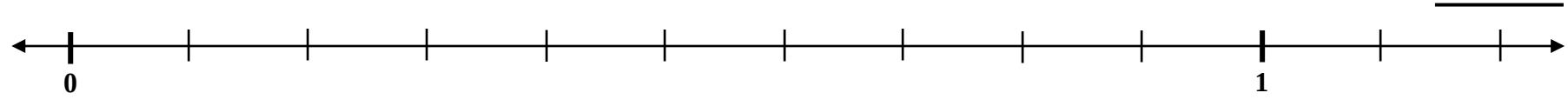
Directions: Determine the fractional division of each number line. Write the answer on the line to the right.

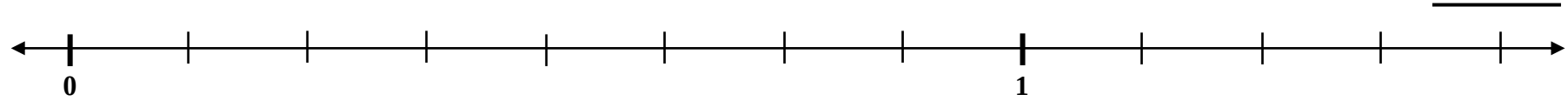
A. 

B. 

C. 

D. 

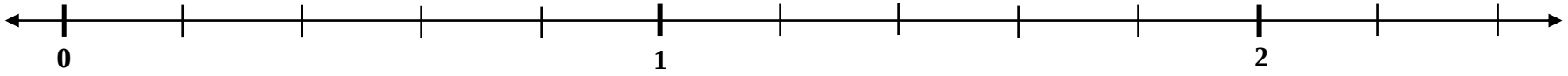
E. 

F. 

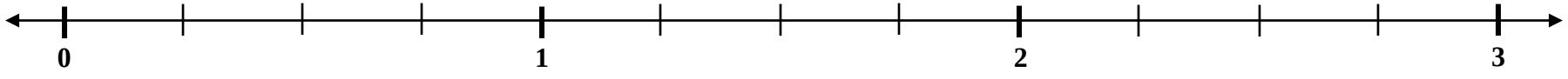
Blank Number Lines – Fraction/Decimal Connection – Version 2

Directions: Determine the fractional division of each number line. Write the answer on the line to the right.

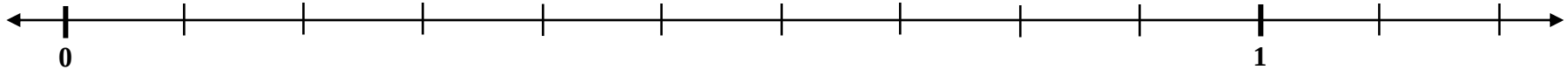
A.



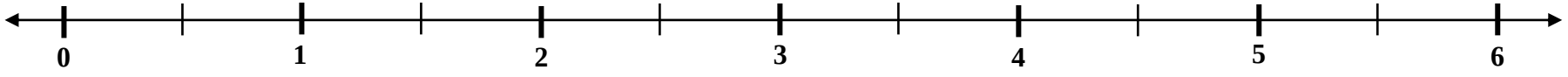
B.



C.



D.



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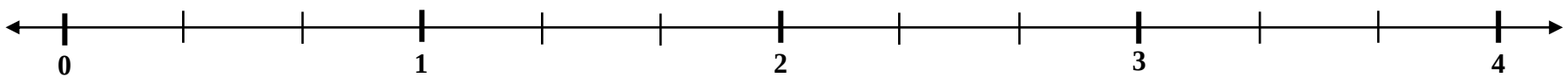

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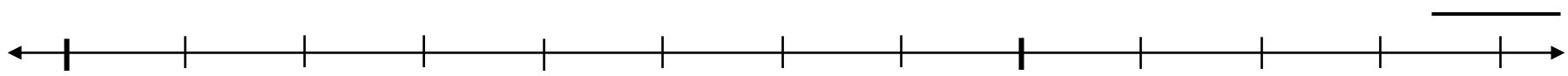
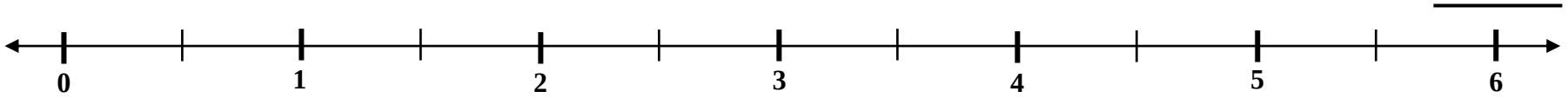
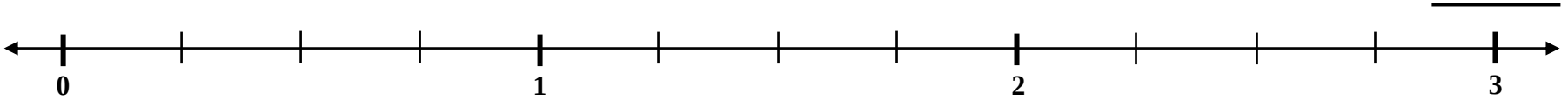
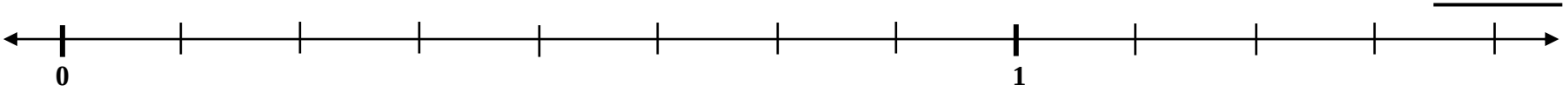


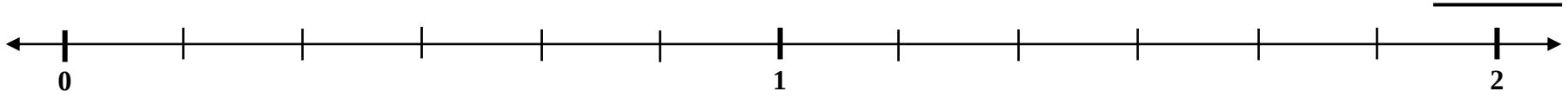
Blank Number Lines – Fraction/Decimal Connection – Version 3

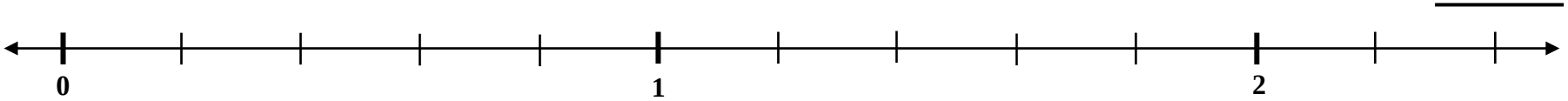
Directions: Determine the fractional division of each number line. Write the answer on the line to the right.

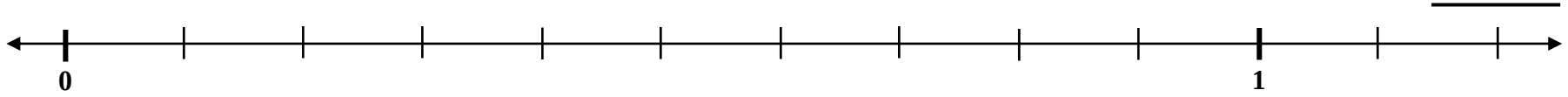
A. 

B. 

C. 

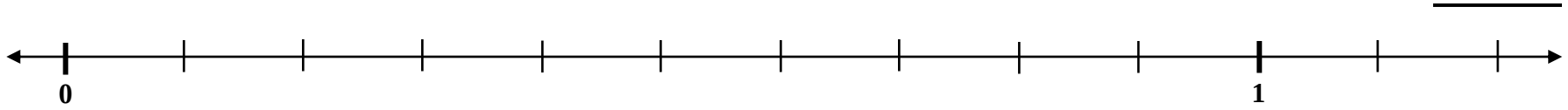
D. 

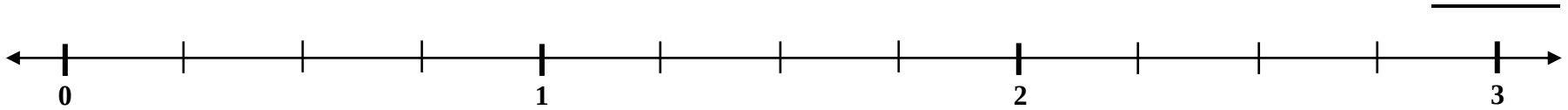
E. 

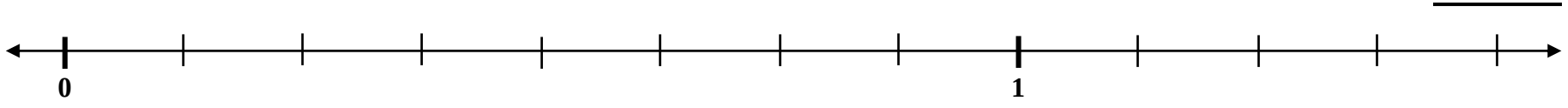
F. 

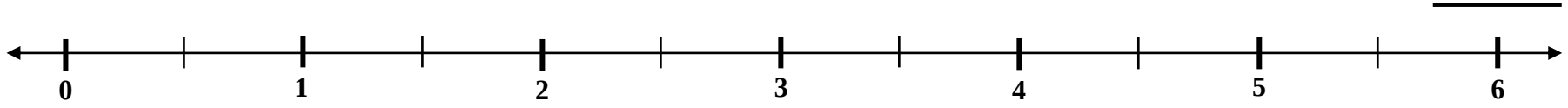
Blank Number Lines – Fraction/Decimal Connection – Version 4

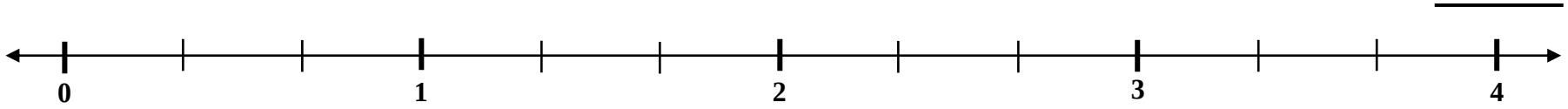
Directions: Determine the fractional division of each number line. Write the answer on the line to the right.

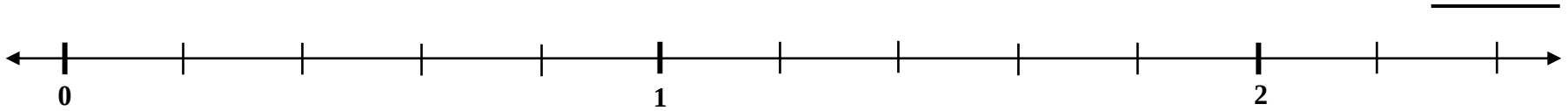
A. 

B. 

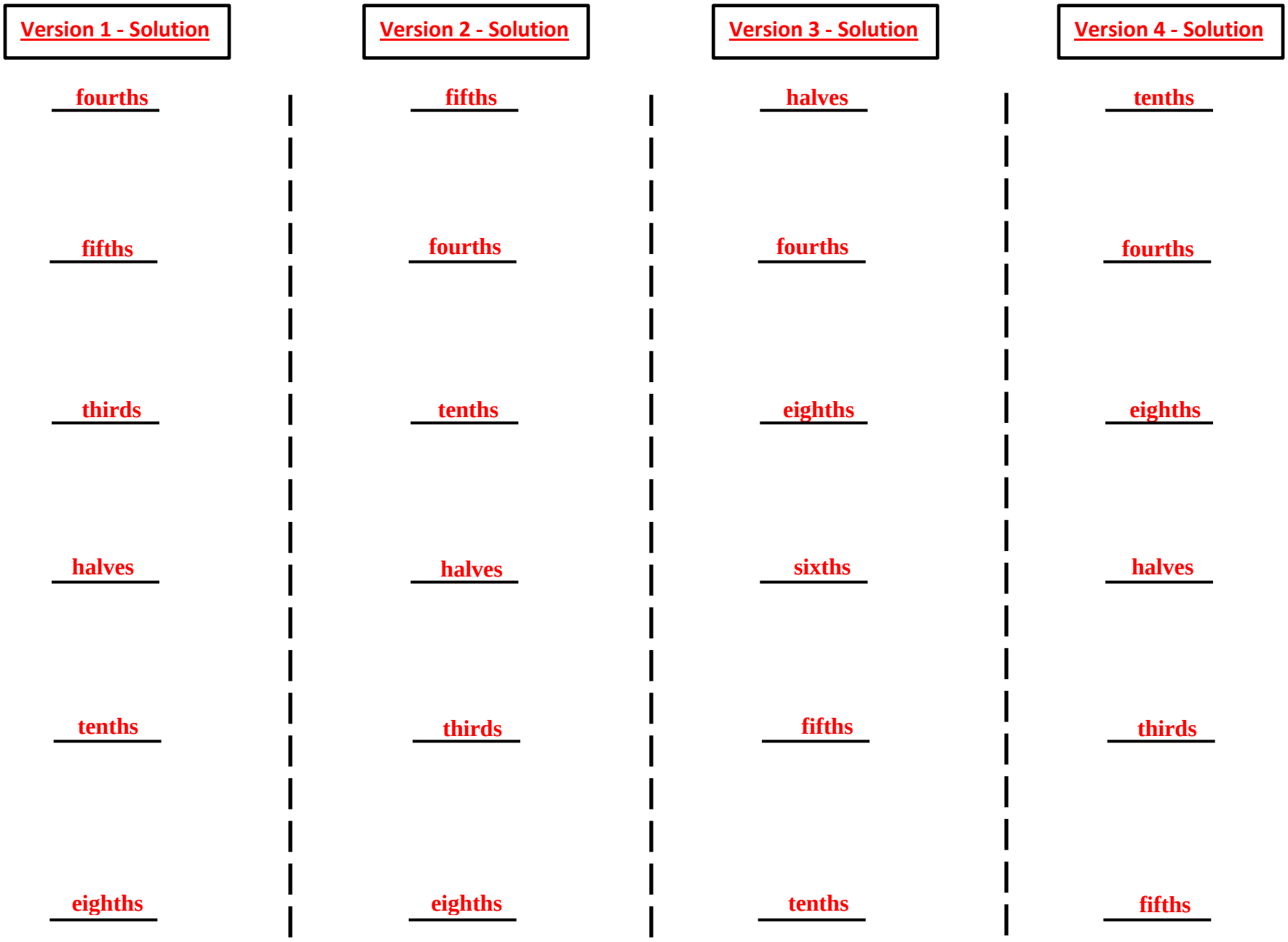
C. 

D. 

E. 

F. 

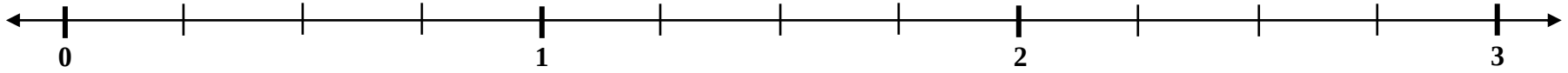
Solutions - Blank Number Lines – Fraction/Decimal Connection – Versions 1 - 4



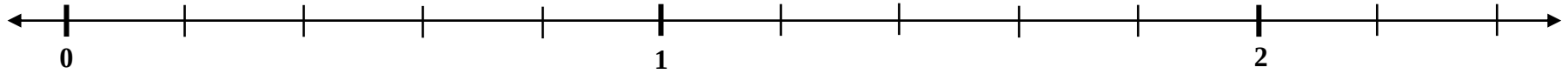
Blank Number Lines – Fraction/Decimal Connection – Version 5

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

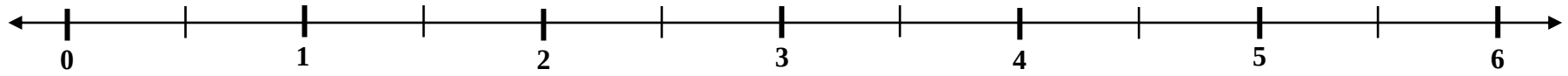
A.



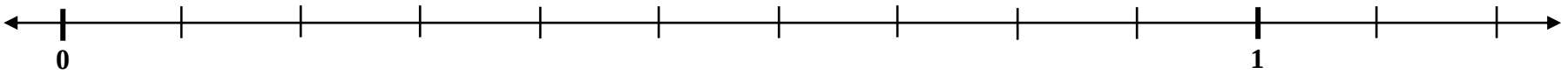
B.



C.



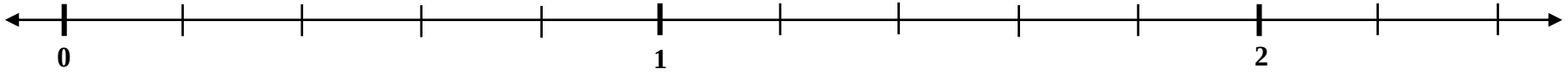
D.



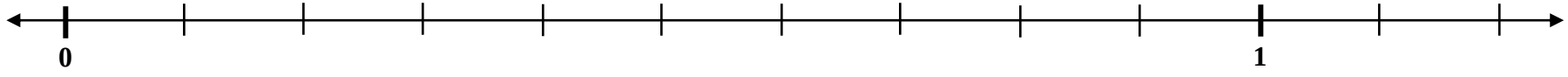
Blank Number Lines – Fraction/Decimal Connection – Version 6

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

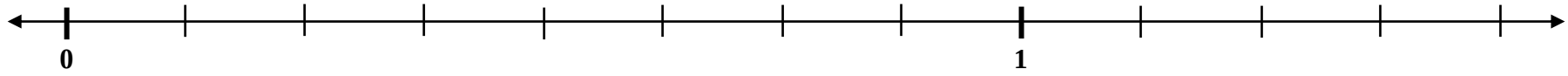
A.



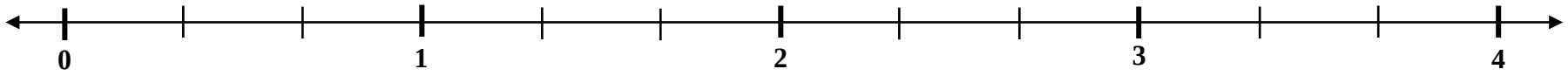
B.



C.



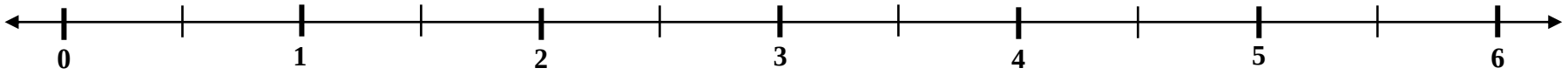
D.



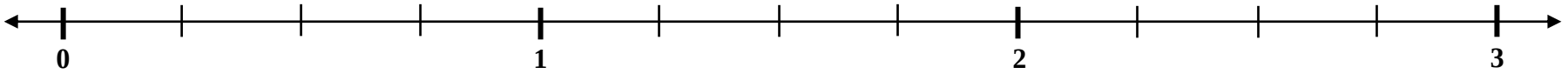
Blank Number Lines – Fraction/Decimal Connection – Version 7

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

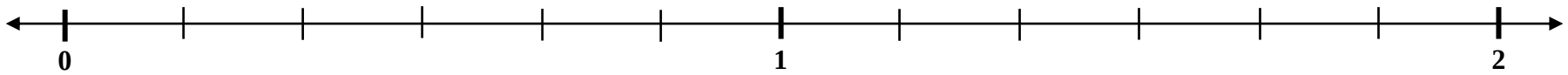
A.



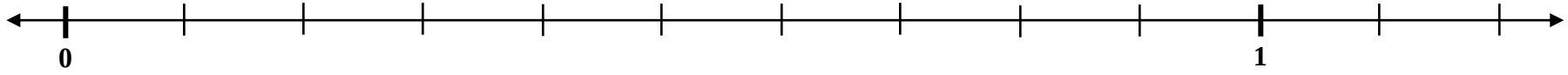
B.



C.



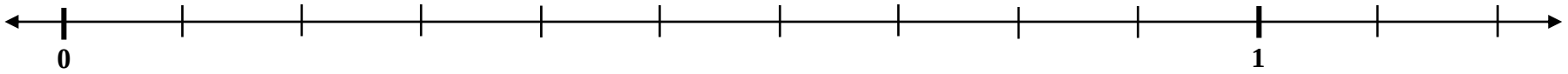
D.



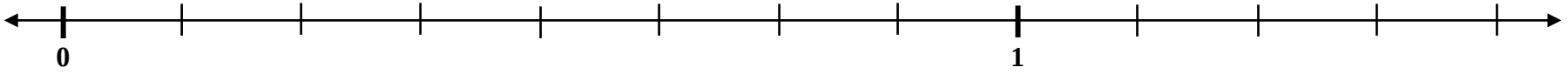
Blank Number Lines – Fraction/Decimal Connection – Version 8

DDIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

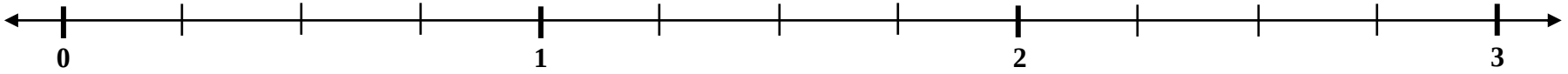
A.



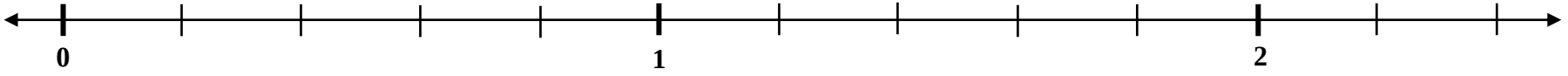
B.



C.



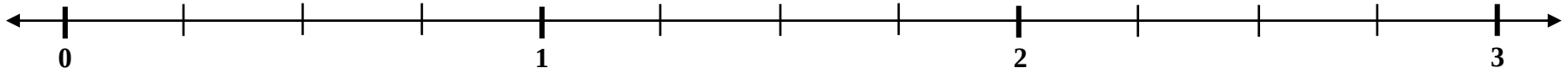
D.



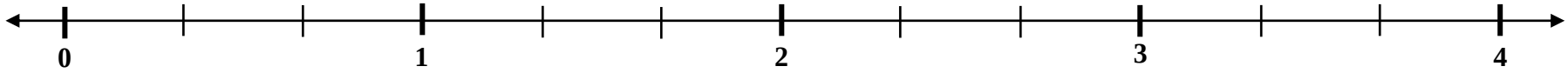
Blank Number Lines – Fraction/Decimal Connection – Version 9

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

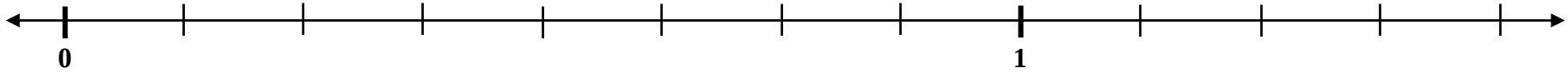
A.



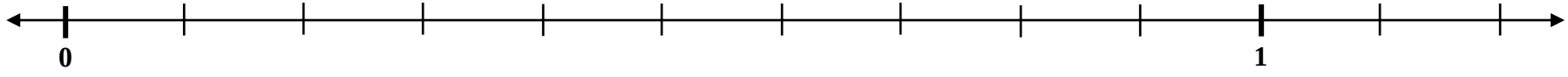
B.



C.



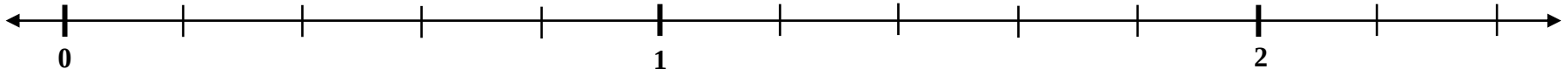
D.



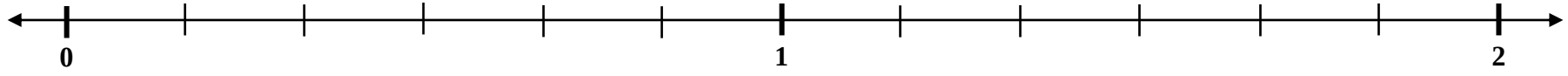
Blank Number Lines – Fraction/Decimal Connection – Version 10

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

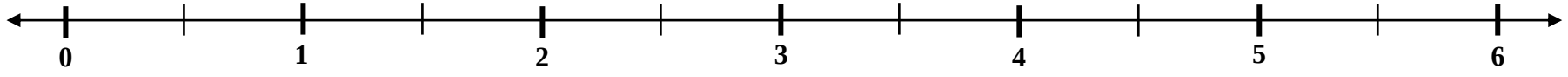
A.



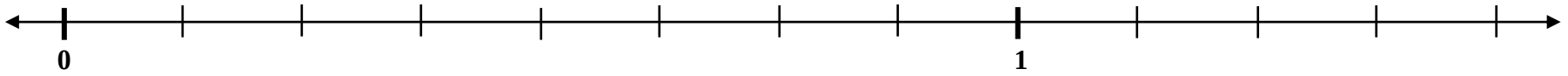
B.



C.

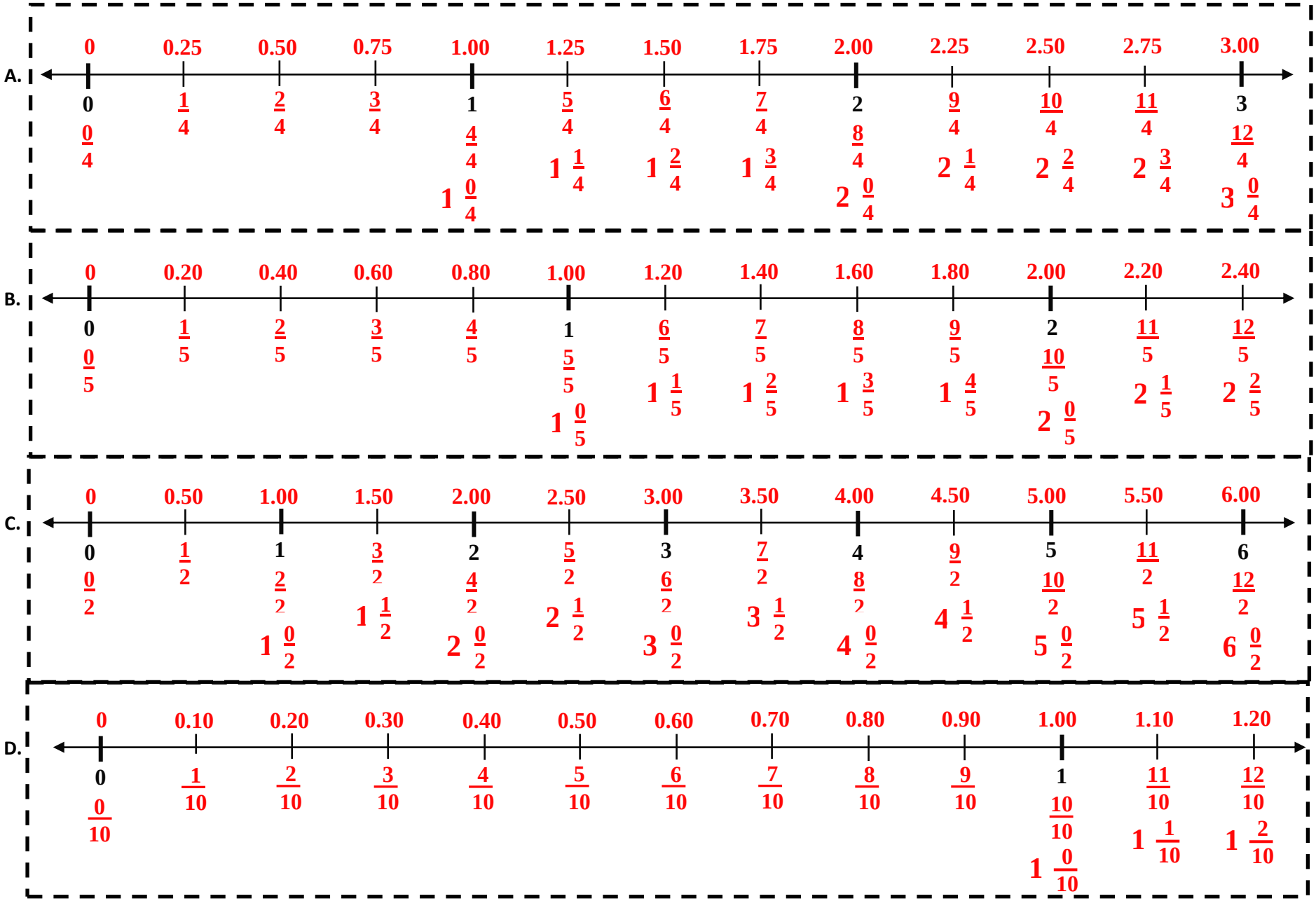


D.



General Solution - Blank Number Lines – Fraction/Decimal Connection – Version 5 - 10

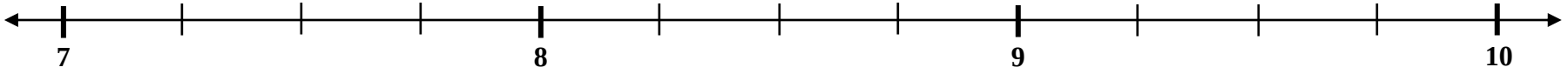
DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.



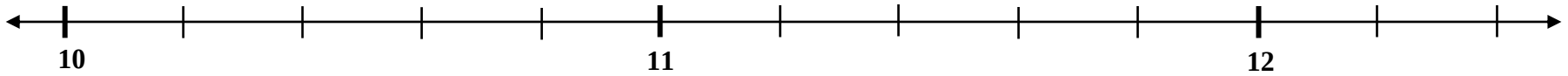
Blank Number Lines – Fraction/Decimal Connection – Version 11

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

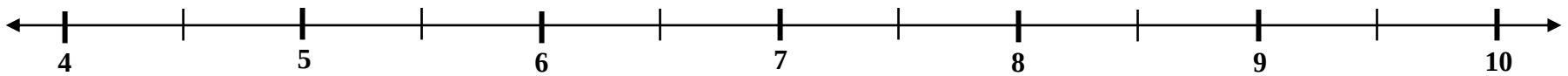
A.



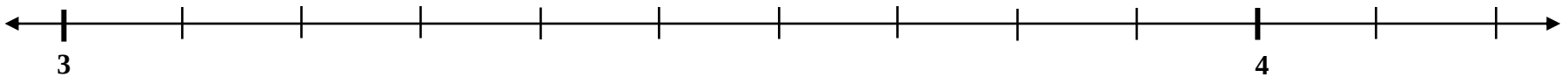
B.



C.



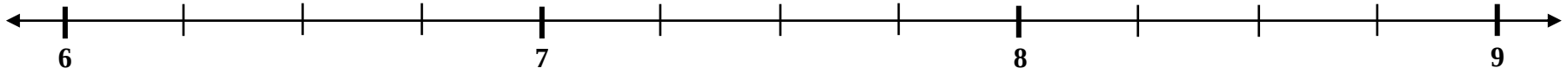
D.



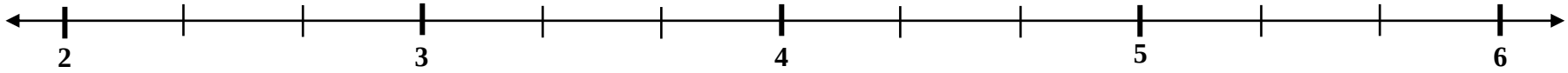
Blank Number Lines – Fraction/Decimal Connection – Version 12

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

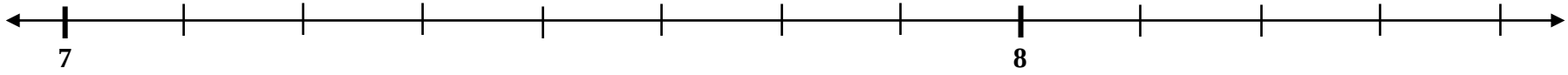
A.



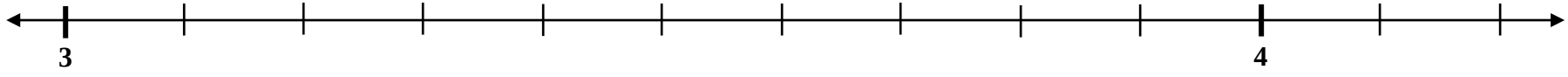
B.



C.

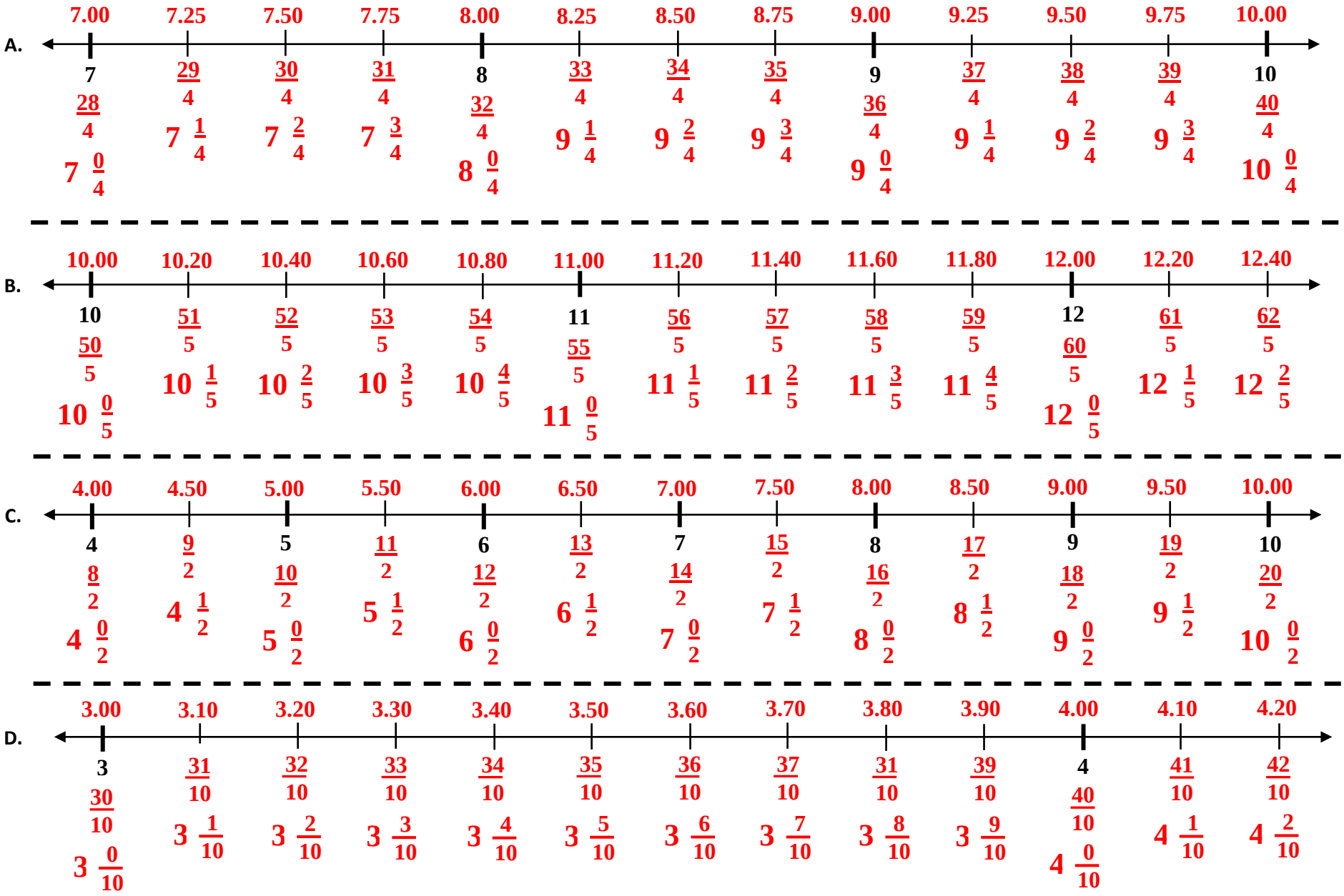


D.



General Solution - Blank Number Lines – Fraction/Decimal Connection – Version 11 - 12

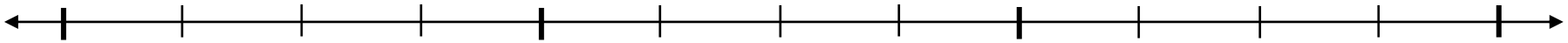
DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.



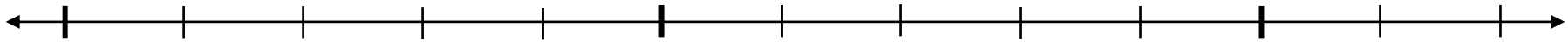
Blank Number Lines – Fraction/Decimal Connection – Version 13

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

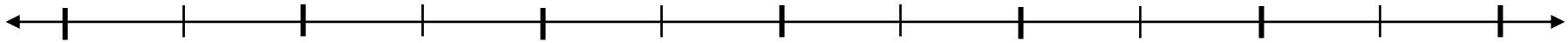
A.



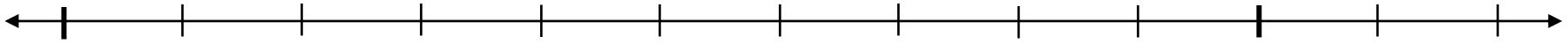
B.



C.



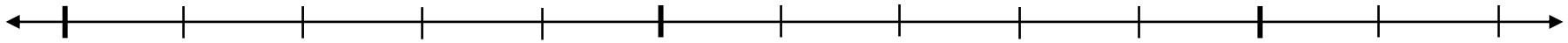
D.



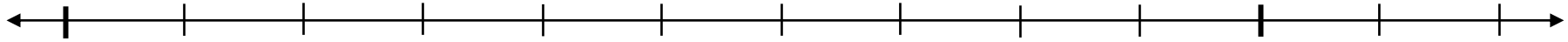
Blank Number Lines – Fraction/Decimal Connection – Version 14

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

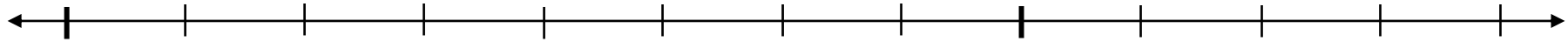
A.



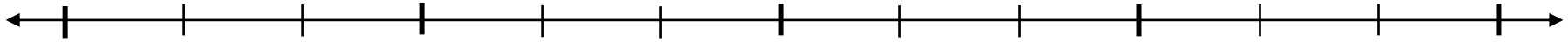
B.



C.



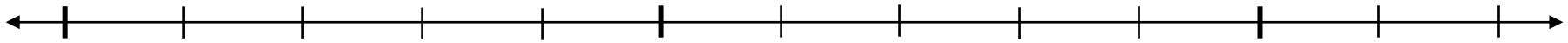
D.



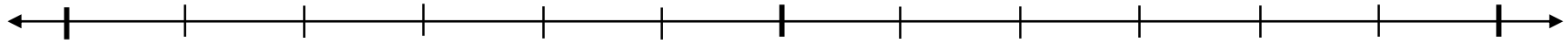
Blank Number Lines – Fraction/Decimal Connection – Version 15

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

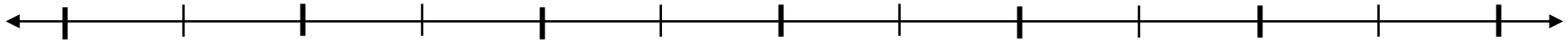
A.



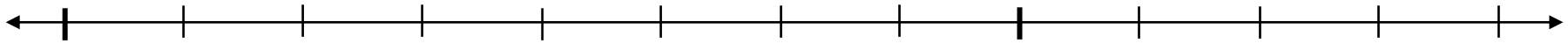
B.



C.



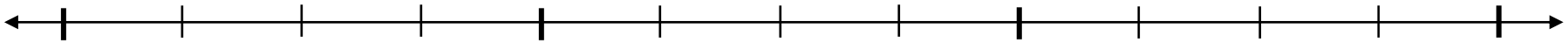
D.



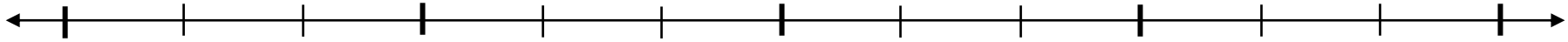
Blank Number Lines – Fraction/Decimal Connection – Version 16

DIRECTIONS: Label each number lines at each division with a *proper or improper fraction* and/or a *mixed number* and/or a *decimal* according to your teacher's instructions.

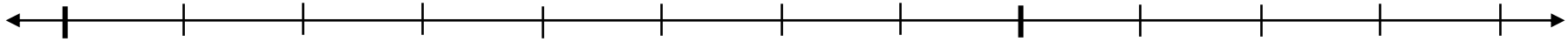
A.



B.



C.



D.