

## GRADE 4 - AMARA SPACED REPETITION SEQUENCING RECOMMENDATIONS

Student mastery of skills is highly dependent on adequate exposure and threshold repetitions. For example, a student must correctly practice a skill over a short period (spaced) a threshold number of times (repetition). Spaced repetition is an efficient and effective means to reach the minimum number of repetitions for each student. General guidelines for the number of repetitions for most students are listed in the following table.

Student Classification	Number of Repetitions for Skill Mastery
Gifted and Talented Identified	1 to 4 repetitions
General Education Students	8 to 18 repetitions
Students Receiving Special Education Services	Varies widely from student to student – Consult each student's Individual Education Plan and follow legal guidelines

Below are the general recommendations for specific skill areas using Amara supplemental daily resources. Due to student dynamics, each classroom's teacher will experience different levels of intervention and presentation of each skill in a spaced repetition process. Therefore, spaced repetition pedagogy is unique for each classroom due to varying student ability in mathematics.

It is suggested that 4 to 9 skills be presented quickly in a highly accountable and engaging teaching mode for approximately 5 to 10 minutes at the onset or the end of the core lesson. *Many skills will require little time for students to master, but others may take more days for students to demonstrate mastery.* The document appears lengthy, but there is an abundance of commentary provided to inform teachers of specific pedagogy and examples. However, it is essential the classroom teacher is highly organized, prepared and plans a spaced repetition process that provides a quick transition between skills. Consistency is the key to success in most human endeavors, and this process is not an exception to that thinking.

The teacher should observe the students to determine which students require more practice until mastery of presented skills is accomplished. After skill mastery is achieved by all students, a teacher can drop that skill from the list below and add a new one. A variation in spaced repetition methodology is when the teacher presents a specific math skill each day until the majority of students have demonstrated skill proficiency. At that point, the teacher can engage students requiring additional practice in a small group setting while the other students complete independent work. Amara's Skill Support Resources are designed for both types of pedagogy and are available on the Amara4education website for purchase. Also, it is recommended that with the use of Formative Loop's ([www.formativeloop.com](http://www.formativeloop.com)) daily numeracy program, students are provided a daily opportunity to master both math fact operational and process skills.

Class-wide accountability and comprehension checks can be done with small white boards, raising hands, a show of fingers to represent number answers, or paper and pencil to name a few. Teachers can position themselves to observe paper-pencil responses and identify specific students that demonstrate a lack of skill proficiency. Finally, please note that the skill list below is a guideline. A teacher should evaluate their students and adjust the list as they feel is appropriate to meet the needs of the students.

### Recommended Skill List – Fourth Grade

- 1.) All Math Fact Operations.** Student proficiency in math fact operations must be pressed for all students. Math fact mastery is essential in arithmetic mathematics for students to master many dependent concepts. One of the greatest gifts a schoolwide numeracy program can provide for students is requiring proficiency of their basic math facts – 100 mixed basic 1-12 math facts in all

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four operations in 5 minutes. This arithmetic objective should be pressed every day of the 4<sup>th</sup> and 5<sup>th</sup> grade elementary years until accomplished. If the student receives special education services, the teacher should follow the legal stipulations in the student's Individual Education Plan (IEP) to the letter. It is the author's professional opinion and experience that all students can master their basic math facts, if the teacher follows methodology that is both pragmatic and goal oriented. The white paper on the Amara website (URL given in footer of this document) will assist the teacher in setting up accelerated math fact methodology, as well as the use of Formative Loop. The Formative Loop Numeracy Program offers a slow build-up via a short daily writing assessment that is key to students ingrain their math facts.

- 2.) **Computing Sums Review.** Begin with simple 2-digit sums. For example, the teacher can begin with addition problems like  $32 + 9 = ?$  and  $45 + 69$ , and transition to 3-digit, 4-digit and 5 digit addition problems. During the spaced repetition process each math period, the teacher should provide 2 to 3 addition problems for review each day until all students demonstrate mastery. With learning opportunities each day, students will become proficient at almost any developmental task.
- 3.) **Computing Differences Review.** Same as 2.) above. Begin with 2-digit differences and transition to 3-digit, 4-digit and 5-digit subtraction problems. NOTE: It is highly recommended that students work at least one subtraction problem of the two or three given where the selected numbers in the minuend require subtraction across zeros and regrouping. For instance,  $504 - 149 = ??$ .
- 4.) **Making 10 and 100 Review.** This exercise is well worth the time investment to build numeracy and prepare students for Making 1 with decimals later in the semester. Students can also use the Making 10 skill to aid in mastering their subtraction facts (1 digit subtracted from 2 digit math facts ( $14 - 5$ )) as well. Begin with number sentences ( $4 + \underline{\quad} = 10$ ) as needed, and transition to mental math using fingers. For example, during spaced repetition process, a teacher can extend 6 fingers and students respond by raising 4 fingers. This method is an easy visual to determine diagnostically what students have mastered the skill as well as those who have not. Third grade students will easily grasp this invaluable Base 10 concept. Transition to an audio reply for students, using appropriate inside student voices. The teacher can say 2, and students respond with 8. Making 100 is relatively easy once the Making 10 skill is mastered. If students are shown that  $2 + 8 = 10$ , then  $20 + \underline{\quad} = 100$  is an easy transition in skill processing for them. After the Making 100 skill is mastered, students can transition to Making 100's by 5's by adding up. For example, the teacher says, Make 100 from 35." The students mentally add-up 35 to 40 for 5, and the 60 more from 40 to 100. Hence, their total is 65. Short, rapid practice is all that is required for numeracy proficiency in any skill to reach the threshold number of repetitions provided in the table on page 1 of this document.
- 5.) **Even and Odd Review.** A teacher will discover that between 10 to 40 percent of their 4<sup>th</sup> grade students have not mastered this primary-aged skill unless spaced repetition was implemented in prior grades. Consequently, it must be reviewed so ALL students are provided skill proficiency in elementary school. Begin with small numbers to show why the ones digit determines if a number is classified as even or odd. It is recommended dividing up the number in 2 equal groups to show an odd number. For example, 6 is an even number due to the fact it can be separated in two equal groups of 3. Seven (7) is an odd number because it cannot. *The divisibility rule of 2 will not make sense to most students at this point in their third grade year.* Use larger numbers until a pattern can be established that even numbers end in 0, 2, 4, 6, 8; and odd numbers in 1, 3, 5, 7, 9. Finally, a valuable tactile method is for students to use their fingers on each hand to determine if a number is even or odd regardless of magnitude. The students match fingers on each hand. For instance, if the number is 6, students should count to 6 alternating extending a thumb/finger on each hand – starting with the thumbs. Then, match the 3 extended fingers on each hand. The two thumbs and extended

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fingers all pair up – with none left over. Hence, the number 6 is even. If the number were 7, one hand would have an extended finger that does not match with a finger on the other hand. Consequently, 7 is classified as an odd number.

- 6.) **Place Value Whole Number Expanded Form.** Begin with 2-digit expansion to show the value of each digit.  $34 = 30 + 4$ . Transition to 3-digit expansion when appropriate in lesson sequencing ( $208 = 200 + 0 + 8$ ). At the appropriate time, expose students to 4 and 5-digit numbers to show the value of each digit. Please note, many students struggle with zeros (0's) in a number. For example,  $34,050 = 30,000 + 4,000 + 0 + 50 + 0 = 30,000 + 4,000 + 50$ . Proceed to the billions place value. One problem a day for a couple weeks, and students will master place value for the rest of their lives.
- 7.) **Place Value Whole Number Expanded to Standard Form:** It is recommended providing an expansion with zeros so students reassemble the number to standard form and demonstrate that they understand zero as a placeholder. For instance and at the appropriate time in the fall semester, the teacher displays/writes on the white board or document camera:  $20,000 + 400 + 1$ . The students respond with the number in standard form: 20,401. Many times, students that have not mastered place value concepts will write this number as 241, omitting the zero placeholders. It is recommended to provide students numeral expansions to the billions place value. Note: The teacher should have these expanded numbers written in advance on a separate piece of paper. Therefore, he/she can quickly place the paper on the document camera and the spaced repetition pedagogy moves rapidly.
- 8.) **Addition and Subtraction Number Line Models Review.** Use whole number lines and the students can review the physical meaning in short learning spurts. The number lines can start with a simple math fact (as needed), but with fourth graders, it is recommended to quickly transition to subtracting 2 digit numbers. For example,  $30 + 20 = 50$  or  $45 - 15 = 20$ . Conversely, students can be given a number line, and they write down the addition or subtraction equation. Use Amara Resources Skill Support for blank number lines practice sheets and addition models that assist in facilitating skill mastery. Once completed with the practice sheet, students can flip the sheet over for more quick repetitions of additional math skills in this spaced repetition list. Minimal review is required, but it is essential that the classroom teacher is assured students understand the physical meaning of addition and subtraction.
- 9.) **Compute missing addends.**  $5 + \underline{\quad} = 7$  or  $\underline{\quad} + 5 = 11$ . Revisit as many times as required for student proficiency. It is also recommended providing students number sentences that are in a different form than they are normally accustomed:  $8 = \underline{\quad} + 3$  or  $12 = 9 + \underline{\quad}$ . Students should be familiar with number sentences besides the standard left to right.
10. **Addition Vocabulary Review:** Addends and sum. Given:  $4 + 5 = 9$ . Indicate the correct mathematical name of each numeral (4 and 5 are addends, and 9 is called the sum).
11. **Commutative Property of Addition.** Students should fully understand that addends can be switched – unchanging the overall addition equation. For instance,  $5 + 4 = 9$  and  $4 + 5 = 9$  are representations of the same math equation. It is highly recommended that the teacher stress that subtraction does **NOT** possess the same commutative property. For instance:  $12 - 5 = 7$ . But,  $5 - 12$  does not equal 7. It equals negative 7 – there are still 7 spaces between 5 and 12, but subtracting a larger number from a smaller one produces a difference that is negative.
12. **Subtraction Vocabulary Review.** Stress the subtraction terms: Minuend, Subtrahend and Difference. Note: Subtrahend can be remembered by students since it is the number Subtracted – both words start with the letter 'S'. For example,  $6 - 4 = 2$ . Since 4 is the numeral subtracted, it is

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also called the subtrahend. Correct vocabulary must be stressed, so students ingrain the math terms into long-term memory.

13. **Making 1,000.** Begin with multiples of 100. For instance,  $300 + \underline{\quad} = 1,000$ . Transition to mental mathematics when the teacher states, “400.” Students respond with 600. Finally, students should be able to add-up from 550 to 1,000. Students can be taught to add 50 from 550 to 600, and another 400 from 600 to 1,000 for 450. It has been the author’s experience that many students may require a horizontal representation of the 50 and 400 jumps. If the teacher writes 550....600...1,000 on the document camera, then students can mentally visualize the 50 and 400 ‘jumps’. Short practice and the students possess another powerful numeracy tool.
14. **Computing an addition or subtraction number sentence in words.** Teacher states, “*What is 5 less than 9?*” or, “*What is 3 more than 18?*” Transition to larger numbers. For example, “*What is 10 less than 105?*” or, “*What is 2 less than 100?*” or, “*What is  $85 - 15 = ?$* ” or “*What is  $80 + 15 = ?$* ”
15. **Fact Family – Addition and Subtraction Quick Review.** Provide the students with three related numbers, and students write the fact family. For instance, teacher can display on the document camera/white board the following: 5, 11 and 6. Students write the 2 addition and 2 subtraction equations.  $5 + 6 = 11$ ;  $6 + 5 = 11$ ;  $11 - 5 = 6$ ;  $11 - 6 = 5$ . Transition to larger numbers: 55, 15 70. Students respond with  $15 + 55 = 70$ ;  $55 + 15 = 70$ ;  $70 - 55 = 15$ ;  $70 - 15 = 55$ .
16. **Computing Halves.** Begin with small numbers. Halve 4 or 8 or 12, etc. Transition to larger numbers. It is recommended that the teacher show pattern numbers at first. For example, half of 6 is 3. Half of 60 equals ??. Half of 600 is ??. Half of 8 is 5. Half of 80 is ??. Half of 800 is ??. Students must understand the patterns of numbers. Finally, transition to mental math once students demonstrate understanding. Hence, students can be given 20 to 30 half problems with only the number to be halved. Students can complete. Student work can be globally checked for accuracy, and the half practice sheet flipped over to the blank side for more work on other skills.
17. **Multiplication Vocabulary.** Factors and Product. Students must know both terms or they will not recognize the terms in print on an assessment. It is recommended to begin by providing a math fact:  $5 \times 6 = 30$ . The students should name the factors and the product. Transition to larger numbers:  $50 \times 7 = 350$ . Multiplication, like addition, possesses the commutative property, so factors can readily be interchanged.
18. **Multiply 2-digit by 1-digit Factors Review.** Begin with simple and easy problems with no carrying unless already covered in core lessons or Amara daily resources.  $12 \times 4$ ;  $6 \times 10$  and  $8 \times 11$ . Transition to multiplication problems that require a carrying of tens.  $18 \times 4$ ;  $25 \times 5$ ; and  $6 \times 48$ . Use the large multiplication model approach so students relate their physical understanding back to the manner they learned place value. See 41. Below for a visual of methodology.
19. **Multiples.** It is highly probable that newly enrolled students have not mastered this numeracy skill. However, students should be able to count (skip count) by 1’s, 2’s, 10’s and 5’s – in this order of presentation/exposure/initial introduction. Include in the space repetition process of each multiple of 3’s, 4’s, 6’s, 7’s, 8’s, 9’s, 11’s and 12’s in small amounts each day to increase numeracy ability. Students will become adept very quickly with minimal practice. It is highly recommended for teachers to include multiples of 100 (0, 100, 200, 300, ...) and 15’s (to 150), 60’s (to 600), 25’s (to 250) and 50’s (to 500). This exercise is invaluable for not only numeracy building, but to use in understanding the physical meaning of multiplication as a repetitive adding concept. It is highly recommended that multiples be practiced in small chunks until they are mastered for numeracy and multiplication math fact proficiency later in the fall semester. The author also suggests sending the multiple exercises in the Amara Skill Resource for homework, so students practice the skill

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independently at home in the evening. Note: There are many numeracy benefits from this type of skip counting practice.

- 20. Clocks. Reading and understanding time Review.** Begin with hours, quarter hours and half-hours. Students should begin by recognizing the time shown on the clock. It is recommended that besides showing 10:30 or 2:00 o'clock form, use terms like "half-past" ; "quarter till" ; "quarter after" so students understand the vernacular of clock time description. Transition to 10 and 5-minute and 1-minute increments when it's appropriate to do so. Many students will struggle with clock times near the hour until they have had sufficient and corrective practice. For example, 5 minutes till 4 (e.g. 3:55). Please note: Three clock examples per spaced repetition session a day for 6 to 7 days, maximum, and all students have mastered this skill.
- 21. Equation Strings:** Use numbers that students can perform mentally. Begin with only a couple numbers, so students can adapt to the process. For example, begin with  $30 + 60 + 10$  and  $25 + 15 - 10$ . The teacher should transition to more numbers, for instance,  $100 + 200 - 50 + 10?$  and  $50 \times 2 + 75 - 5?$  Note: Depending on the class situation, it may be necessary to initially write and display the equations on the document camera, and then transition toward mental computations.
- 22. Comparing whole numbers with  $<$ ,  $>$  or  $=$  Review.** Begin with small numbers and transition to 3- and 4-digit numbers. This review should be rapid at this age, but it ensures the teacher that all students possess this skill.
- 23. Rounding – Whole Number Review.** The teacher should review rounding to the nearest 10, 100, 1,000 and 10,000. Quick exercises each day – observing students that may struggle until all students master these concepts. Note: If a student has trouble counting past 109 which is common with about 10 percent of students in Title 1 elementary schools, the teacher may need to use whole number lines in a small setting and the student physically label the numbers to rectify that skill deficiency. Note: It is recommended that the teacher specify that the half-way point (i.e. a number ending in 5, 50, 500, 5,000) is arbitrarily selected to round UP not DOWN. For example, the number 65 is exactly 5 units away from both 60 and 70; however, human convention dictates that the number 65 rounds to 70 and NOT 60. Students should know that this rule is one of human choice/convention not mathematical purpose.
- 24. Adding/Subtracting - Estimation.** Practice one to two problems a day until mastered through the ten thousands place value. Note: Students must be told, estimate the addends or subtrahend/minuend FIRST, then add or subtract. Too many times students add or subtract and then round the sum or difference. This is incorrect and defeats the purpose of estimating a difference or sum.
- 25. Time Units:** Time equivalencies must be structurally taught to students. For example, students should know the following time facts with automaticity: 60 minutes in one hour, 24 hours in one day, 7 days in a week, about 30 days in one month, 12 months in one year, 365 days in one year, 10 years in a decade, 10 decades in a century, 100 years in a century, 10 centuries in a millennium and 1,000 years in a millennium. Students should also know that currently this is the third millennium, 21<sup>st</sup> century and that decades are commonly referred and referenced as the 70's, 80's, 90's, etc. Note: 1.) the length of a day (24 hours) is determined by the time it takes the Earth to make one rotation on its axis, 2.) the length of a month (approximately 30 days) is the time it takes the moon to make one revolution around the earth, 3.) one year is determined by the time it takes the earth to make one complete revolution around the sun (365 days). One week has 7 days – this fact arises that when the days were originally named millenniums ago, the ancients only knew of 7 celestial objects in the sky (Sun – Sunday), (Moon – Monday), (Tuesday – Mars), (Wednesday – Mercury), (Thursday – Jupiter), (Friday – Venus) and (Saturday – Saturn). The modern English translations from Latin obscure this fact, but both French and Spanish retains the Latin origination much clearer in the

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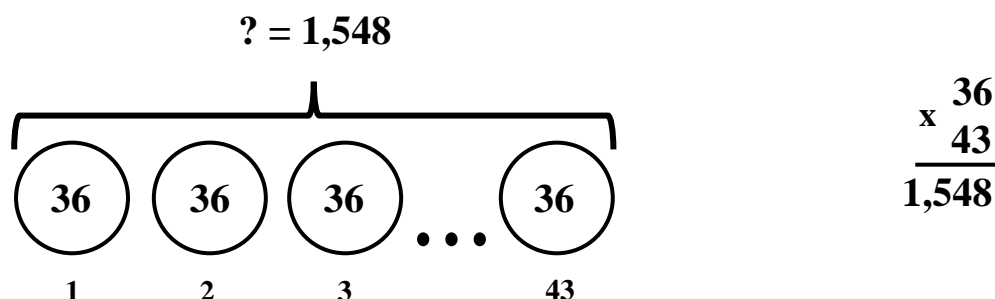
spelling of their days of the week. For instance, “Tuesday” is martes in Spanish and Mardi in French – FYI only.) Students should know that everyday names and facts of time that define our world were not randomly selected.

- 26. Multiplication models (Group, Array, Area and Number Lines) Review.** It is imperative that students are aware of the physical meaning of multiplication. The teacher can show these models quickly until the students realize they are representations of a math fact with only a different model. For instance, the teacher can show three circles of 4 items in each circle (group model) OR a number line with three equal jumps of 4 OR an array of 3 stars by 4 stars OR a grid or 3 columns by 4 columns. Quick demonstrations where the children write down the math fact of  $3 \times 4 = 12$ . Multiplication is nothing more than repetitive adding no matter what the model. Hence, the multiple work that students complete early on in the semester is invaluable to building numeracy and physical understanding of multiplication concepts and models. Students should also create an array, group or area model when provided a simple multiplication math fact (e.g.  $4 \times 2 = 8$  – students draw an array or group model.) A short white paper entitled “Large Whole Number Multiplication and Division Diagrams” shows each multiplication model. It is available for free download at the website below.
- 27. Elapsed Time Review.** The student must first be able to read and understand clock time. It is recommended that the teacher show two clocks (quarter hours, half hour or hour time only). Students calculate the elapsed time. Problem types should be simple– no problems involving a transition from AM to PM. For example, one clock may show 3:45 and a second clock 4:15. Students compute the elapsed time. Note: it is helpful to require students to learn the first five multiples of 15 separately (e.g. 0, 15, 30, 45, 60) as a separate skill in spaced repetition. In five days, all students will have mastered multiples of 15, and elapsed time is much easier since students previously have mastered skip counting (multiples) of 5 and 10s. Finally, as students become more proficient, the teacher can provide simple word/verbal problems involving elapsed time. For instance, “*Luz left home at 4:30 and she returned at 5:45. How long was she gone?*” Transition to more difficult elapsed times. **These types of skills are exposure dependent.** If students struggle with the concept of elapsed time, they may be unable to read the time on a clock or they cannot count and add-up the time to the next hour (e.g. Making 60 from a whole number between 1 and 59) and/or compute hours using multiples of 60. Another dependent skill with elapsed time is converting minutes to equivalent hours and/or minutes. If a teacher practices this skill a threshold number of times, students will master it.
- 28. Doubling Whole Numbers** – It is recommended to begin with small whole numbers: 2, 3, 4, 5, etc. Include doubling tens so that students see the pattern – double 2 = 4; double 20 = 40; double 4 = 8; double 40 = 80. Transition to 100’s and 1,000’s so students can extend skill base to larger numbers based on small whole number patterns. Note: Doubling numbers like 15, 25, 55, etc. require threshold practice to be successful. However, students should master doubling small whole numbers and tens prior to doubling intermediate numbers like 15, 25, 55, etc.
- 29. Large Whole Number Group Multiplication Model** – The students should ‘own’ a mental multiplication whole number model. This skill allows students to dissect and parse a word problem to a diagram model. First, begin with students completing the diagram model. After this aspect is mastered, students should practice drawing the model, given a multiplication equation. Frequently, students do not transition from small whole number equations to larger whole number equations. For example, students may know that  $5 \times 6 = 30$ , and the equations physical meaning is 5 equal groups of 6 or 6 equal groups of 5 for a total of 30. But, students do not apply that same thinking toward a larger equation such as  $47 \times 9$ . The whole number group model promotes that all multiplication (and division) equations represent the same physical model regardless of the magnitude of the factors. The group model is thoroughly expatiated in a short white paper entitled “Large Whole Number

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Multiplication and Division Diagrams. It is available for free download at the website provided in the footer. Note: Align 2-digit multiplication to core curriculum sequencing of math program used at the classroom or grade level.

**Example:** Compute and show the meaning of  $36 \times 43 = ?$ .



43 equal groups with 36 in each group yields 1,548. Or,  $43 \times 36 = 1,548$

- 30. Multiplication Vocabulary.** Factors and Product. Students must know both terms. It is recommended to begin by providing a math fact:  $5 \times 6 = 30$ . The students should name the factors and the product. Transition to larger numbers:  $50 \times 7 = 350$ . Also, show the math fact of one of the models described above, and the students write the equation and label the factors and product as well.
- 31. Verbal Multiplication Modeling Practice** – The teacher provides one or two quick word problems to students. For instance, “John saved 68 dollars each month for 27 months. How much money did John save?”  
The students should correctly draw the group model diagram and solve the problem. This daily practice until mastered will afford students to recognize only the product is missing as well as physical understanding of the mathematics.
- 32. Estimation of Multiplication Review.** Review approximation of products. Note: Students may need to be reminded it is always a KNOWN, BASIC math fact that must be used to compute the factors’ product. Hence, on a multi-digit factor multiplied by a single digit, the single digit is NOT rounded. For instance,  $78 \times 4$  rounds to  $80 \times 4$  to take advantage of math fact and reasonable product.
- 33. Verbal Half and Double Practice** – As in 29.) above, the teacher should provide simple word problems that students can quickly solve. For instance, “Jasper is 30 years old. Montee is half Jasper’s age. How old is Montee?” Or, “Andreas is 20 years old. Olga is twice as old as Andreas, and Janna is half as old as Andreas. How old are Olga and Janna?” If the computational skill sets of doubles and halves were previously mastered, students will be very adept at working these problem types in only 3 to 5 days.
- 34. Review 2D Polygons (and a circle)** – The teacher can review this quickly. Review Polygons: triangle, quadrilateral types - (parallelogram, trapezoid, rectangle, rhombus, square), pentagon, hexagon, octagon. Include regular hexagon and octagon where all sides and angles are equivalent in measure. Review terminology: parallel, perpendicular, vertices. Students should possess immediate recall of these figures.
- 35. Division Vocabulary Review.** Quotient, Dividend and Divisor. It is recommended to begin by providing a math fact:  $48 \div 8 = 6$ . The students should name each number in the equation. Transition to larger numbers:  $560 \div 8 = 70$ .
- 36. Division Facts - Missing Factors** – This skill should be combined with larger whole numbers as well. For instance,  $3 \times \underline{\quad} = 27$ ;  $3 \times \underline{\quad} = 270$ . The missing factor problem in reality is a division

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problem – computing either the divisor or quotient. The three (3) sheets entitled, “Find the Missing Factor” are one of the most pragmatic process sheets for students to master their division facts. Those sheets are available in the Amara4Education Skill Support Resource Packets for both third and fourth grades. Those sheets will ensure that students master missing factor and division facts simultaneously.

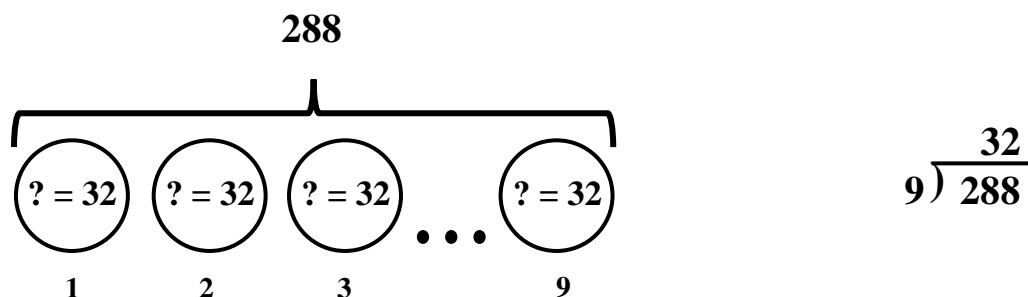
- 37. Fact Family – Multiplication and Division Quick Review** – As with addition and subtraction, provide the students with three related numbers, and students write the fact family. For instance, teacher writes on the document camera: 6, 18 and 3. Students write the 2 multiplication and 2 division equations.  $3 \times 6 = 18$ ;  $6 \times 3 = 18$ ;  $18 \div 3 = 6$ ;  $18 \div 6 = 3$ . Transition to larger numbers: 32, 3, 96. Students respond with  $32 \times 3 = 96$ ;  $3 \times 32 = 96$ ;  $96 \div 3 = 32$ ;  $96 \div 32 = 3$ .
- 38. Basic Fraction Review** – The teacher should conduct a quick review of fraction and fraction terminology of denominator and numerator. This review should afford student review of halves, thirds, quarters/fourths, fifths, sixths, eighths and tenths. Note: The student should also be shown improper fractions of  $\frac{2}{2}$ ,  $\frac{3}{3}$ ,  $\frac{4}{4}$ , etc. and understand the fractional word form (i.e. three-thirds). The student should not only review a polygon or circle separated into equal parts, but multiple similar objects shaded as a unit. For example, if the student is given 7 stars and 3 are shaded, then the fraction of stars is  $\frac{3}{7}$ .
- Group Form Example:** a set of 5 birds with 4 shaded.  $\frac{4}{5}$  is the fraction.
- Diagram Form Example:** a circle of rectangle divided into fourths with a couple segments or all segments shaded. Students respond with the fractional amount. Note: Students should be familiar with the terms quarter, half, thirds, halves, fourths, fifths, eighths, sixths, tenths in word form. Example three-fifths ( $\frac{3}{5}$ ) or four-fourths ( $\frac{4}{4}$ ) or two-tenths ( $\frac{2}{10}$ ).
- 39. Base 10 Place Value Expansion.** Students should be exposed to a daily diet of two problems until this skill is mastered. Begin with the two-digit numbers then transition into 3, 4, 5 and 6 digit numbers. Students should be given a two-digit number and write the number in expanded base 10 form. For example:  $34 = (3 \times 10) + (4 \times 1)$  or  $409 = (4 \times 100) + (0 \times 10) + (9 \times 1)$ .
- 40. Measurement Daily Review** – There is a daily warm-up (free) digital download entitled, “*4<sup>th</sup>-8<sup>th</sup> Grade Daily Measurement Warm-up*” at the website address in the footer. The daily resource covers mass/weight, length and volume/capacity for both customary and metric units. This warm-up covers the basics with short application of each measurement type. The resource can be completed independently by students in five minutes each day and quickly checked. The  $\frac{1}{3}$  sheet of paper can then be flipped over and the teacher can continue with a spaced repetition instructional system and cover more skills. This level of efficiency is highly beneficial to preserving instructional minutes in the math block regardless if the math block is 60 minutes or 90 minutes.
- 41. Large Whole Number Group Division Model** – This is the same group model as the multiplication group model, but for division. As with multiplication, the students should ‘own’ a mental division whole number model. This skill allows students to dissect and parse a word problem to a diagram model. First, begin with students completing the diagram model. After this aspect is mastered, students should practice drawing the model, given a division equation. As with the multiplication explanation, students do not transition from small whole number equations to larger whole number equations. For example, students may know that  $32 \div 8 = 4$ , and the equations physical meaning is 32 objects separated into 8 equal groups yields 4 items in each group. But, students do not apply that same thinking toward a larger equation such as  $320 \div 80 = 4$ . The whole number group model promotes that all division equations represent the same physical model regardless of the magnitude of the dividend and divisor. The group model is thoroughly expatiated in a short white paper entitled



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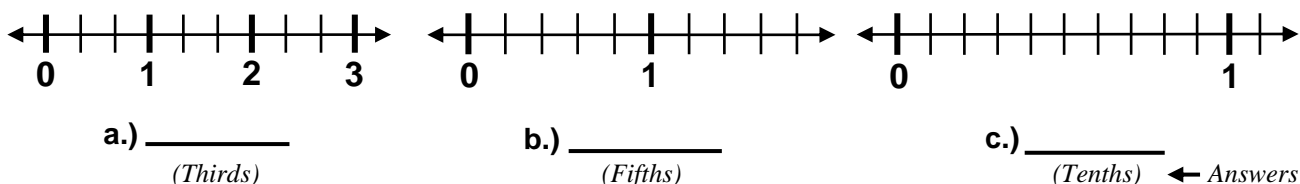
**Example:** Compute and show the meaning of  $288 \div 9 = ?$ .



288 separated into 9 equal groups yields 32 in each group. Or,  $288 \div 9 = 32$ .

- 42. Determining the Denominator of a Fractional Number Line** – It is highly recommended that the correct vocabulary of proper fraction, improper fraction, mixed number, numerator and denominator be used when describing fractions, or students will not recognize these same mathematical terms in print form on a state assessment. Note: Students should be given a blank figure or diagram and immediately be able to compute the denominator. Students must also be reminded that fractions always possess equal segments of division. A critically important introduction in fraction concepts are fractional number lines. Students should be given a blank fractional number line with only the whole numbers listed (see below). Students should be able to determine the fraction that describes the number line by counting the ‘**SPACES**’ and NOT the **LINES** between the whole numbers. Students can visually show the teacher with their fingers the denominator of each number line as he or she points to the number line displayed on the document camera (i.e. students extend 3 fingers if the number line is in thirds). The number line facsimile should be prepared in advance for the rapid review of the fractional number lines and denominator identification. Focus on halves, thirds, fourths, fifths, eighths and tenths.

**Example:** Write the **denominator** of the fractional number lines on the line provided.



### Vocabulary Note:

Proper fractions are less than 1 whole or 1 (e.g.  $\frac{1}{2}$ ,  $\frac{7}{8}$ ,  $\frac{2}{4}$ ).

Improper fractions are **greater than or equal** to 1 whole (e.g.  $\frac{4}{3}$ ,  $\frac{2}{2}$ ,  $\frac{6}{6}$ ,  $\frac{9}{5}$ ).

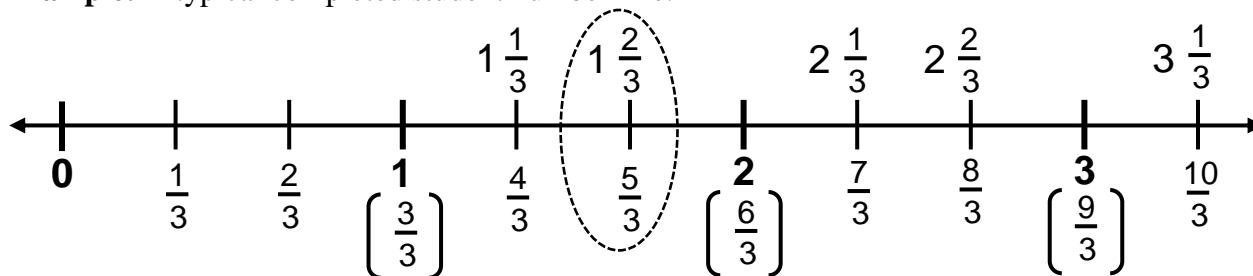
Mixed Numbers possess both a whole number and a proper fraction (e.g.  $1\frac{1}{2}$ ,  $2\frac{3}{4}$ )

- 43. Completing a fractional number line with fractions and mixed numbers** – Students should be able to rapidly write the fractional number lines for  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{8}$  and  $\frac{1}{10}$ . If given one or two fractional number lines per day, students will be adept at this skill in 5 days. Students should begin by determining the fractional number line’s denominator (see 42 above). Then, complete all proper fractions, improper fractions and mixed numbers on the number line. After students complete the

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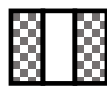
exercise, the teacher should ask students, “Locate  $\frac{2}{3}$  on the number line. Draw a picture of that fraction.” Students can use a circle, or a square divided into thirds and shade two segments for the numerator. The teacher should repeat that exercise for an improper fraction and a mixed number. The teacher can also ask students to draw a picture of an improper fraction and mixed number AT THE SAME point on the number line and show that fraction and improper fraction are equivalent. It is also recommended that the teacher require students to write the equality (i.e.  $1\frac{2}{3} = \frac{5}{3}$ ).

**Example:** A typical completed student number line.

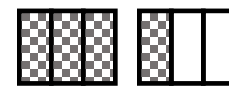


Students can create a Proper Fraction and Mixed Number.

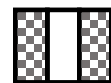
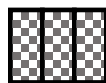
$$\frac{2}{3} =$$



$$1\frac{1}{3} =$$



The following equivalency can be established using the number line:



$$\Rightarrow \frac{5}{3} = 1\frac{2}{3}$$

**Important Note:** A student’s ability to complete these number lines will connect fractions and mixed numbers to a fixed location in space as primary grade teachers analogously use whole number lines. This exercise will prevent these fraction concepts from “floating” in space. **Within a week or two of quick daily repetitions, students will be remarkably adept at this valuable skill.** Critical pedagogy that is a game-changer in a student’s understanding of fractions and mixed numbers!!!

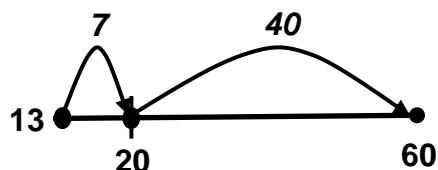
- 44. Verbal Division Modeling Practice** – As with multiplication verbal modeling, the teacher provides one or two quick word problems to students. For instance, “John saved 180 dollars last year over 9 months. How much money did John save each month?”

The students should correctly draw the group model diagram and solve the problem. This daily practice until mastered will afford students to recognize either the divisor or the quotient is missing as well as physical understanding of the mathematics.

- 45. Make 60** – Making 60 is an important numeracy skill for students to own in reference to elapsed time and reading a clock. Students can learn to ‘add-up’ with minimal practice. Begin with the tens until mastered. For instance, Given 20 minutes after ‘2 o’clock.’  $20 + \underline{40} = 60$ . Then, transition to the fives. For instance, Given 25 minutes after ‘5 o’clock.’ 25 to 30 is 5 minutes. 30 to 60 is 30 minutes. Therefore, 35 minutes (5 + 30). Repeat from the 1’s. For example, 27 minutes after ‘8 o’clock’. 7 to 10 is 3 minutes. 30 to 60 is 30 minutes. Hence, 33 minutes (3 + 30). It is highly recommended that students practice *Making 10* to mastery as well practice *Making 60* with 5’s and 10’s prior to learning the process of adding up, so there is automaticity in their numeracy. In doing so, students can focus on the process and not have their CPU used up in computations. The practice sheets are available in the Amara Skill Set Resources. It is also recommended to use a linear diagram to physically show students the mathematical process. The example below illustrates this method.

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Making 60 Diagram:



Add -up minutes from 13 to 60:  $7 + 40 = 47 \text{ minutes}$

- 46. Complete Simple Equations with a Remainder** – *These problems should be simple* to show the concept on basic math facts, so students understand the remainder concept prior to engagement in more difficult problems. Show two to four equations per day until students demonstrate mental proficiency. Note: Remind students what the remainder means – that the remainder shows how many ‘objects’ could not go into an equal group. Note: The remainder is **ALWAYS** less than the divisor, or there are enough objects to make another equal group (i.e. Remainder < Divisor). Note: It is also recommended that students practice checking the dividend via multiplication and adding.

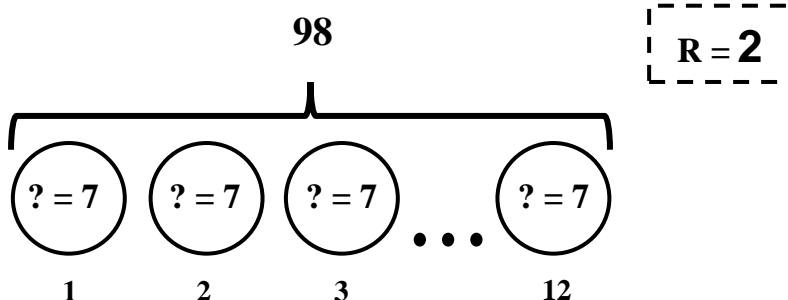
Example: Students complete the equation by **mentally** computing the remainder. Check by multiplying and adding.

$$\begin{array}{r} 7 \text{ R } ? \\ 3 \overline{) 22} \end{array}$$

Check:  $(3 \times 7) + 1 = 22$

- 47. Large Whole Number Group Division Model WITH Remainder** – This model adds in the remainder into the visual solution. Understanding the physical meaning of remainders is often challenging for many elementary students. Hence, the visual group model eliminates much of these issues. Students should also check their work by multiplying the divisor by the quotient and add the remainder to verify the dividend, and that their division computation work is correct. The white paper referenced in number 41 above clearly expatiates this methodology. Note: It is imperative to stress the *remainder is always less than the divisor*, or another equal group can be made (e.g. in the example below, the remainder is equal to 2 and the divisor is 12. Hence,  $12 > 2$ .)

**Example:** Compute and show the meaning of  $98 \div 12 = ?$ .



$$\begin{array}{r} 7 \text{ R } 2 \\ 12 \overline{) 98} \end{array}$$

98 separated into 12 equal groups yields 7 in each group with a remainder of 2 left over. The computation work can be checked as follows:  $(12 \times 9) + 2 = 98$ . Note: Use parenthesis, so students become accustomed that parenthesis means “Do my math first.”

- 48. Comparing Fractions** ( $<$ ,  $>$ ,  $=$ ) – There are a couple methods for comparing fractions. The first method does not promote physical understanding. The student is multiplying each fraction by a common multiple of the product of both denominators. For instance, comparing two fractions in the example below. The student is multiplying each fraction by the two denominators (i.e. 5 and 4) and canceling out the denominator on side. For instance,  $\frac{2}{5} \times (4)(5)$  cancels out the  $\frac{5}{5}$  (which equals 1 whole) and leaves only  $2 \times 4 = 8$ . Of course, the product of 4 and 5 (i.e. 20) is a common multiple

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of 5 and 4. For that reason, the student will always obtain the correct comparison. However, it is not a comparative method that promotes understanding.

$$\frac{2}{5} \bigcirc \frac{1}{4} \quad \Rightarrow \quad \overset{\textcircled{8}}{\frac{2}{5}} \quad \frac{2}{5} \times \frac{1}{4} \quad \overset{\textcircled{5}}{\frac{1}{4}} \quad \Rightarrow \quad \frac{2}{5} > \frac{1}{4}$$

A more complete and recommended method in fraction comparison is a stepped process. **First**, the student must be competent and adept at calculating halves of whole numbers. In short, the teacher should require students to be able to mentally compute half of most whole numbers that are divisible by 2 (e.g. 2, 4, 6, 8, 10, 12, 16, 30, 40, etc.). There are practice sheets in the Amara4education Skill Support Resource Packet for halves. **Second**, students need to understand magnitudes of fraction – meaning the relative comparing a numerator and denominator. There are three evaluation areas that must be taught to mastery – closest to zero (0), half and closest to a Whole (1). Hence, if the numerator is very small compared to the denominator, the fraction is closest to zero (0) – for instance,  $\frac{1}{8}$ . If the numerator is approximately half of the denominator, then the fraction is about  $\frac{1}{2}$ . It is also recommended that students evaluate if the fraction is greater than  $\frac{1}{2}$  or less than  $\frac{1}{2}$  to assist in comparing specific fractions. For instance,  $\frac{7}{16}$  – since  $\frac{8}{16}$  is half, then  $\frac{7}{16}$  must be less than one half. If the numerator is close in value to the denominator, then the fraction is about 1 whole (i.e.  $\frac{8}{10}$ ). **Third**, if the numerators are the same, then compare the denominators. The larger denominator will always be smaller –  $\frac{4}{7}$  and  $\frac{4}{10}$ . If the denominators are the same, then the fraction with the larger numerator will always be larger –  $\frac{5}{8}$  and  $\frac{6}{8}$ .

**Example:** Compare the fractions,  $\frac{6}{14}$  and  $\frac{9}{16}$  to determine which fraction is larger.

Analysis: Since half of 14 is 7,  $\frac{6}{14}$  must be less than  $\frac{1}{2}$ . Since half of 16 is 8,  $\frac{9}{16}$  must be larger than  $\frac{1}{2}$ . Hence  $\frac{9}{16} > \frac{6}{14}$ .

Note: Many state standards stipulate that students should estimate to the  $\frac{1}{4}$  and  $\frac{3}{4}$  points as well.

This is also done, if and only if the teacher requires that level of mental numeracy from their students.

- 49. Qualitative Perimeter and Area Review** – It is important that students readily understand the conceptual differences between perimeter and area. This process is easy for a teacher to do. A teacher can have students hold a textbook or piece of paper in one hand and use their other hand to illustrate the textbook/paper's perimeter by running their hand around the outside edge of the book/paper. They can also visually display the concept of area by running their free hand over the cover of the book/paper. As with all human endeavors, this exercise will take several days to master; however, this is the design of spaced repetition to insure mastery. Note: It is recommended the teacher use different planar objects and not the same textbook or paper each day. Note: This method is a tactile method of learning which assists in the long-term memory of the task.

### Vocabulary Note:

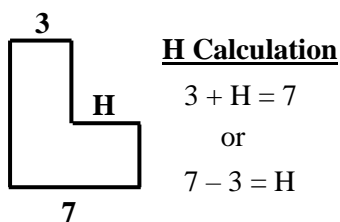
Area is the inside space of a 2-dimensional polygon. Its units are square units/inches/centimeters.

Perimeter is the distance around the outside of a 2 dimensional polygon. Its units are linear units in millimeters, centimeters or inches. Also, the prefix 'peri' means **around**. And, 'meter' means **measure**. Hence, perimeter is defined as '**measure around**.'

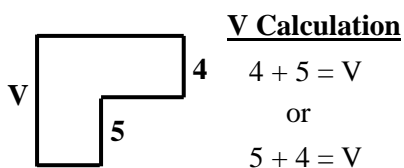
- 50. Find the Missing Side(s) of an L-Polygon Review.** The teacher can present an L-Shaped Polygon (Hexagon) and the students find the missing sides. Use the Amara Practice Sheets on this skill for

## GRADE 4 - AMARA SPACED REPETITION SEQUENCING RECOMMENDATIONS

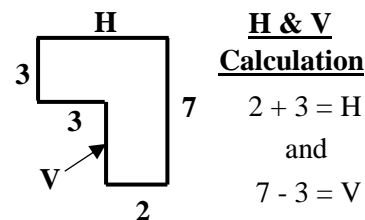
homework, after this skill has been presented in class. It should be noted that this is a challenging task for almost all students in this grade; however, with sound pedagogy and short practice sessions, students master it. It is also an important skill to understand for computing perimeter and area problems. The teacher should focus only on the horizontal dimensions initially with vertical dimensions NOT present on the figure (first example on the left). After the horizontal process is mastered, do the same for the vertical dimensions (middle example). Finally, combined both missing horizontal and vertical dimensions on the same diagram (example below on right). Students master this skill with correct practice and consistent pedagogical instruction.



**Horizontal L-Shape**



**Vertical L-Shape**

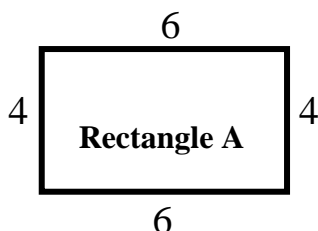


**Horizontal-Vertical L-Shape**

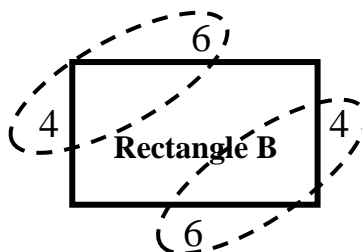
- 51. Perimeter and Area Computations – Simple Grid – Review as Needed.** Students have a gridded rectangle or square. Students count the squares inside the parallelogram and compute the area. Students can also count the squares on the outside edge of the polygon and compute the perimeter. It is recommended the students place a check (✓) on the inside of the rectangle when computing the area, and the place a check (✓) on the outside edge when calculating the perimeter so they count correctly.
- 52. Perimeter and Area Computations – Parallelograms and Polygons – without grid – Use only area calculations on parallelograms, but **perimeter** computations can be for any polygon from a triangle to an octagon.** Provide two problems each day for a week and a half, and the students will own this content.
- 53. Perimeter and Area Computations – L-Shaped Grid – Review as needed.** Similar procedure to above. Students should be given a simple L-Shaped Grid, and the students count to compute the area and perimeter. Easy examples to promote the concept should be provided. Note: Require the students to use checks (✓) for each to minimize and eliminate errors in counting.
- 54. Perimeter and Area Computations – L-Shape without grid.** If given all the sides and the students are not required to calculate a missing side as shown above, this is relatively easy for children to compute that possess math fact competency. The **perimeter** computations are a straightforward adding problem of six numbers. However, on the **area** computations, the students should separate the L-Shape polygon into two (2) separate rectangles. Then, they should compute the area of each rectangle and add the two products together to find the total area of the L-shaped polygon. The most difficult aspect of the area computation is for students choosing the correct adjacent sides to multiply and compute the area of each rectangle. If this is the case, then require students to lightly shade one rectangle and crosshatch the other rectangle so they may better visualize the two discrete rectangular areas of the L-shape.
- 55. Find the missing side in a perimeter or area problem.** Students should practice this example until they are familiar with the concept and adept at the computations. With perimeter, students should be able to compute the missing side regardless of polygon. For area, restrict the computations to parallelograms.

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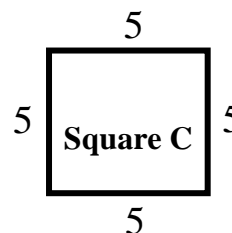
- 56. Parallelogram Perimeter computations using multiplication.** Students often become accustomed to only adding when computing the perimeter of rectangles and squares. The perimeter formula that involves multiplication often confuses them – Perimeter of a rectangle =  $2 \times (L + W)$  or a square =  $4 \times (s)$ . However, if the students calculate the perimeter by addition only, and then use the formula with a visual group model, they readily understand it. After that, only several days practice in the spaced repetition process and they become highly proficient in either computation. See examples below.



**Perimeter of Rect. A**  
 $P = 4 + 6 + 4 + 6 = 20$



**Perimeter of Rect. B**  
 $P = 2 \times (4 + 6) = 20$   
 Note: 2 equal groups of 10.



**Perimeter of Square C**  
 $P = 5 + 5 + 5 + 5 = 20$   
 $P = 4 \times (5) = 20$   
 Note: 4 equal groups of 5.

- 57. 3-Dimensional or Space Figure and attributes review.** Teachers should review the names of all space figures from the primary grades: triangular prism, triangular pyramid, cube, rectangular prism, rectangular pyramid, pentagonal prism, hexagonal prism, octagonal prism, and sphere. Students should be able to count the correct number of the faces, vertices and edges on each space figure. Students often struggle between the appearance of a prism and a pyramid. A pyramid comes to a single point from each side. Hence, if the figure has a single vertex, it is a pyramid, not a prism.
- 58. Make 1.0** – This skill is a tremendous numeracy builder. It takes an analogous advantage of the Base 10 aspect of whole numbers in making 10 and 100. Begin this exercise with Making 1.0 with tenths. Note: Relate to dimes to promote physical understanding (e.g. 1 dime = 0.10; 2 dimes = 0.20, etc.) The equations are finding a missing addend. Hence, a typical equation is  $0.8 + \underline{\hspace{1cm}} = 1.0$ ; and  $\underline{\hspace{1cm}} + 0.30 = 1.0$ . Use the Amara Skill Support Resource Sheets to quiz the students to press for speed, and it is recommended that the students write the '0' in 0.30 or 0.3. After students have mastered the tenths, the hundredths are extremely easy. It is recommended that only the midpoints be used, unless the students are highly numeric in their mathematical abilities. For example, the equations should look like:  $0.95 + \underline{\hspace{1cm}} = 1.00$  and  $\underline{\hspace{1cm}} + 0.75 = 1.00$ . Add-up to the nearest tenth then to 1.00. For example,  $0.35 + \underline{\hspace{1cm}} = 1.00$ . Add 0.35 to 0.40 for 0.05. Then, 0.40 to 1.00 is 0.60. The total is  $0.05 + 0.60 = 0.65$ . With small amounts of practice the students become adept quickly. Again, the practice sheets are in the Amara Skill Support Resource Packet.
- 59. Multiples of Decimals** – A major issue with students is understanding decimals. An easy numeracy solution is quick multiples (skip counting) of decimals, as was with whole numbers. The decimals the teacher should focus are the following: 1¢, 2¢, 10¢, 5¢, 25¢, 50¢ **OR** 0.01, 0.02, 0.10, 0.05, 0.25, 0.50. **Note:** Equate the decimal skip counting to money for physical understanding. Note: When students are counting by 0.01 or 0.02, they should become accustomed to the magnitude of decimals as they reach ten cents (0.10). Note: Remind students that  $0.95 + \underline{\hspace{1cm}} = 1.00$  is 0.05 (i.e. 5¢ not 0.50 or 50¢).
- 60. Converting a Decimal to an Equivalent Proper Fraction/Mixed Number** – This is an easy means for students to convert between decimals and proper fractions or mixed numbers. It requires them to

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place an imaginary '1' **directly** under the decimal point. Then, students add zeros under each number **BEHIND** the decimal point, including zeros. The visual examples below should show this method more clearly.

**Examples:** Convert the decimals - 0.05, 0.302 and 0.7 to *equivalent proper fractions*.

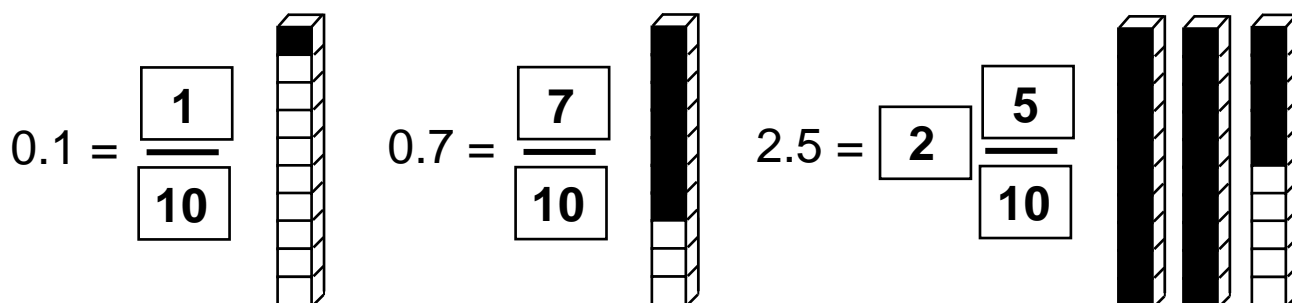
$$0.05 \Rightarrow \frac{0.05}{1\text{00}} = \frac{5}{100} \quad 0.302 \Rightarrow \frac{0.302}{1\text{000}} = \frac{302}{1,000} \quad 0.7 \Rightarrow \frac{0.7}{1\text{0}} = \frac{7}{10}$$

**Examples:** Convert the decimals – 1.04, 3.008 and 5.2 to *equivalent mixed numbers*.

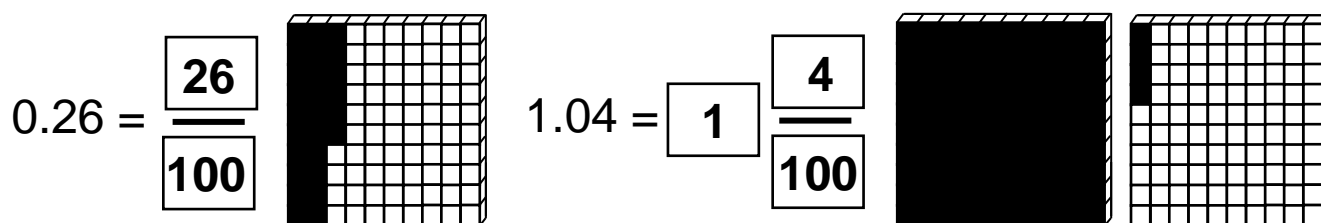
$$1.04 \Rightarrow \frac{1.04}{1\text{00}} = 1 \frac{4}{100} \quad 6.008 \Rightarrow \frac{6.008}{1\text{000}} = 6 \frac{8}{1,000} \quad 5.2 \Rightarrow \frac{5.2}{1\text{0}} = 5 \frac{2}{10}$$

**61. Shading hundredth and tenth blocks to physical meaning of Proper Fractions and Mixed numbers to and from Decimals** – Students must be able to mentally transition between decimals and proper fractions and mixed numbers. In order to do this with meaning, students should practice with 10 blocks and 100 block models. Use Amara Skill Support Resources for downloadable and printable practice sheets for modeling decimals, proper fractions and mixed number physical meaning.

**Examples (10 blocks):** Find the equivalent proper or mixed number. Then, shade an equivalent proper fractions or mixed numbers.



**Examples (100 blocks):** Find the equivalent proper or mixed number. Then, shade an equivalent proper fractions or mixed numbers.



**Note:** Use 10 and 100 blocks to show physical meaning by shading proper/improper fractions and mixed numbers.

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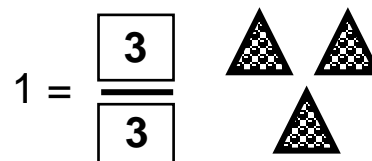
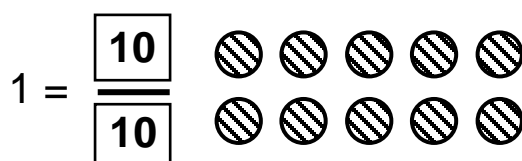
- 62. Adding zeros to decimals – Not changing its value** – Students should see that adding a zero to a decimal does not alter its value (e.g.  $\frac{9}{10} = \frac{90}{100}$  or with decimals:  $0.9 = 0.90$ ). Teachers can use a 100 block to demonstrate that 9 columns out of 10 is the same area as 90 blocks out of 100. The teacher can also use money. For example, “9 dime out of 10 equals 90 cents, and 90 pennies out of 100 is also equivalent to 90 cents.” Hence,  $0.9 = 0.90$ . It is recommended that the teacher repeat examples until this mathematical validity is ingrained. **Note:** If the students are mathematically advanced, the teacher can use equivalent fractions to illustrate the equivalency by showing that  $\frac{9}{10} = \frac{90}{100}$  via multiplying an improper fraction of  $\frac{10}{10} = 1$  (i.e. identity property of multiplication – multiplying a number by 1). Mathematical proof of equivalent fractions to decimal is shown below.

$$1 = \frac{10}{10} \quad ; \quad \frac{9}{10} \times \frac{10}{10} = \frac{90}{100} \quad ; \quad \frac{9}{10} = \frac{90}{100} \quad ; \quad 0.9 = 0.90$$

- 63. Understanding One Whole of a Proper Fraction – Various Mediums** – There are several numeracy exercises that are paramount for elementary students to comprehend to solidify the conceptual and physical sense of proper fractions. These elements should be sequenced to students in a structured means as provided below.

- a.) Students must realize that 1 whole is equivalent to an **improper fraction** with an equal numerator and denominator. The teacher should provide repeated examples until mastered, “If there are five circles and all five circles are shaded, then the improper fraction is equal to  $\frac{5}{5}$  or 1 whole.” **Note:** Draw a quick picture to show the physical meaning. It is recommended that students draw the picture of the improper fraction by shading the squares, circles or ovals that they sketch as their representation. Stress the vocabulary term “improper fraction” where the numerator is equal or greater than the denominator – provide examples ( $\frac{5}{5}$ ;  $\frac{6}{2}$ ;  $\frac{8}{5}$ ;  $\frac{2}{2}$ ; etc.)

**Examples:**

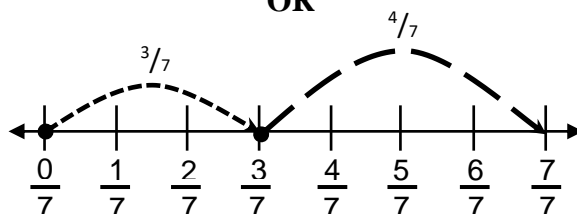


- b.) Students are given simple proper fraction ‘word problems’ and they draw a picture or complete an easily constructed number line to find the solution – and show the physical meaning of the components.

**Example:** John has  $\frac{3}{7}$  of his homework completed. What fractional element remains until John finishes his homework?



OR



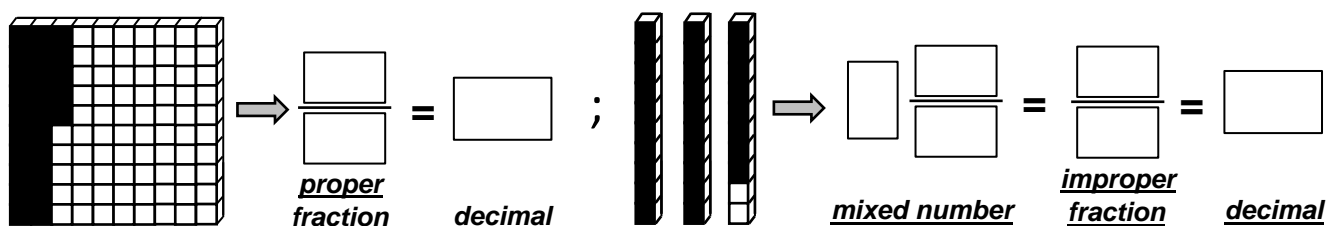
**Student explanation:**  $\frac{3}{7}$  is completed. Hence,  $\frac{4}{7}$  remains to finish for John to finish his homework.



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- c.) **Making 1 - Proper Fractions** – Students mentally Make 1 given a proper fraction. For instance, given  $\frac{3}{5}$ . The student will write the proper fraction  $\frac{2}{5}$  to sum both proper fractions to make the improper fraction of  $\frac{5}{5}$ . Use Amara resource practice sheets in Amara Skill Support Resource.
- d.) **Connecting decimal, proper fractions, improper fractions and mixed number equivalencies** – Students must connect the different fractional and decimal elements so they do not “float” as independent quantities. Teachers should use 10 blocks and 100 block diagrams and students may practice writing these elements from a shaded diagram.

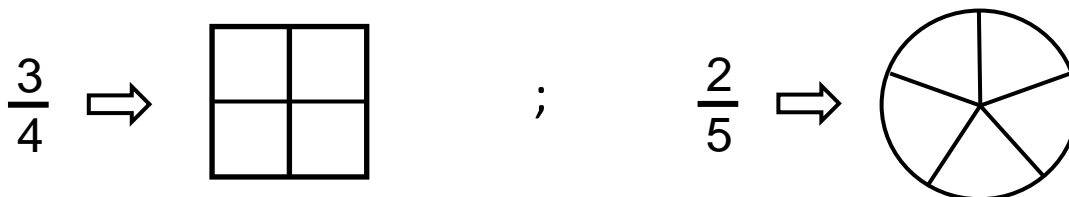
**Examples:**



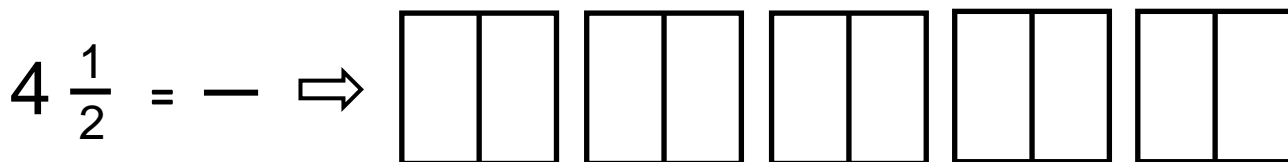
64. **Mixed Number and Improper Number Shading (Not Base 10 denominators)** – Students need to discern between mixed numbers, proper and improper fractions without a base 10 denominator. For example – students will become adept at shading  $1\frac{3}{4}$ ,  $\frac{6}{5}$  and  $\frac{1}{3}$  with minimal but threshold practice. Use Amara skill resource sheets to minimize preparation of student modeling practice sheets. **Note:** Students will learn to convert these elements to equivalent decimals in 5<sup>th</sup> and 6<sup>th</sup> grades, but they are often confused that they cannot change these elements to decimals as they did with base 10 denominators (i.e. 10, 100 and 1,000). For the advanced student, the teacher can practice this process by conveying that a proper or improper fraction always represents two things: part of a group and a division problem. Hence, the teacher can show a student that  $\frac{2}{5}$  is either 1.) two out of five objects and/or 2.) 2 divided by 5 to obtain a decimal:  $2 \div 5 = 5\overline{)2}$  equals 0.4.

**Examples:**

Shade figure to match the proper fraction on the left.



Shade the mixed number on the left and write an equivalent improper fraction.

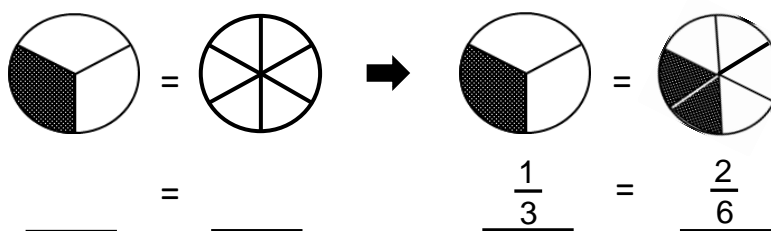


**NOTE:** It is highly recommended to repeat the denominator number line recognition and fractional number line activities described in 42.) and 43.) above so students ingrain and connect all fractional elements in a singular context. This process provides a mental schema and connects proper and improper fractions as well as mixed numbers in one contextual medium.

## GRADE 4 - AMARA SPACED REPETITION SEQUENCING RECOMMENDATIONS

**65. Equivalent Fractions** – Students are exposed to a different couple of versions of equivalent fractions. First, they need a physical model of fraction bars/strips, parallel fractional number lines or an equal area model for two fractions. **Note:** The teacher should consider using a tactile model of fraction pieces based on a diagnostic metric of their students’ initial understanding. Second, students compute equal fractions given 1.) a missing numerator or denominator or 2.) students use an LCD to compute an equivalent fraction. Third, students compute equivalent fractions to reduce or simplify a fraction to lowest terms. Use Amara Skill Support Resources. Examples of each are shown below.

- a.) Students compute equivalent fractions using an equal area approach to understand the physical meaning. Example: Shade the proper fractions so they are equivalent. Write the fractions on the line under each fraction.



- b.) Students compute equivalent fractions by determining a missing numerator/denominator or a like fraction based on a LCD. (**Note:** It is imperative that students understand they are multiplying or dividing the fractions by 1 whole – the identity property of multiplication/division.)

$$\frac{3}{4} \xrightarrow{\times 2} \frac{6}{8} ; \frac{1}{2} = \frac{4}{8} ; \frac{3}{5} = \frac{12}{20} ; \frac{10}{15} = \frac{2}{3}$$

*Multiplying by 1 =  $\frac{2}{2}$*

- c.) Students compute equivalent fractions by reducing a fraction to lowest terms. (**Note:** It is imperative that students understand they are dividing the fraction by 1 whole – the identity property of division (e.g.  $1 = \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}$ , etc.). Finally, a fraction is in lowest terms or simplest form if the only if the common factor between the numerator and denominator is 1.

Factor Strings:

5: (1, 5)

20: (1, 2, 4, 5, 10, 20) **GCF = 5**

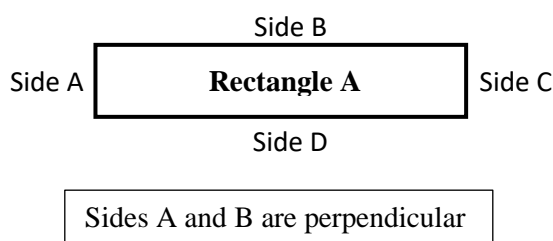
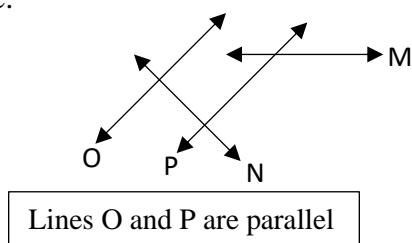
$$\frac{5}{20} \xrightarrow{\div 5} \frac{1}{4}$$

Lowest term check - Factor Strings:

1: (1)

4: (1, 2, 4) **GCF = 1 – LT checks!**

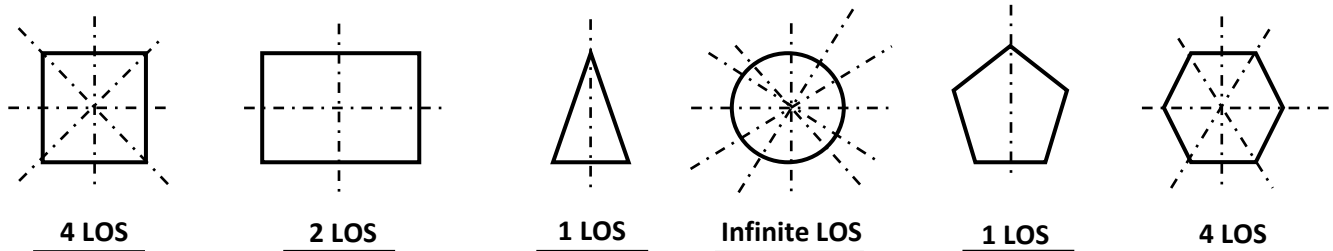
**66. Parallel and Perpendicular Lines.** The word ‘parallel’ has two l’s in its name adjacent to each other. Hence, students can remember parallel lines by this mnemonic. Students should be give parallel and perpendicular lines in a diagram, and then name two of each as an example. Extend the concept into parallelograms. Students can write two lines as parallel and/or two lines as perpendicular. For example:



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**67. Lines of Symmetry (Review)** – Lines of symmetry is a useful skill set for students to develop. Students should possess the ability to view a two dimensional object and count or note the lines of symmetry on the polygon or circle. Students should be aware of diagonal lines of symmetry. Note: If students need a tactile manipulative, create shapes out of construction paper and students can fold the object along the lines of symmetry (LOS) to form two equal/congruent shapes.

**Examples:** Draw the lines of symmetry (LOS) for a square, rectangle, triangle, circle pentagon and a regular hexagon.



**68. Place Value (Word) Form** – This exercise completes the place value processing skill. Students should be able to write numbers in word form to the hundred thousands place. Students may need reminding to use a hyphen between the numbers less than a hundred (e.g. 49 = forty-nine). Students should spell the words correctly. If students struggle with the correct spelling of words in number form, there is a practice sheet in the Amara Skill Support Resource to assist students. It is recommended that this process begin with two-digit numbers, and build each day with a couple examples until students are competent spelling 5 and 6 digit numbers. *The only time the word ‘and’ is used in place value word form is when students are writing decimals, since the word ‘and’ represents the decimal point.*

**69. Factor Strings** – With consistent practice and Formative Loop, students can learn math facts to mastery, but they also can master factor strings for all numbers from 1 through 60. In addition to general numeracy benefits, factor string mastery are of assistance in reducing proper fractions to lowest terms. An easy means for students to ingrain and become proficient with factor strings to mastery is using the Compression Method. The ‘*Compression Method*’ is duly named because factors are computed outward - inward (  $\rightarrow \leftarrow$  ). Hence, students utilize basic divisibility rules and math fact knowledge to write ALL the factors for a given number (from 1 to 60) from the extremities of the brackets to the factor string’s center. The author has written a free download (white paper) that expounds on the Compression Method in much more detail, and it is available at the website address listed in the footer. **Note:** This approach affords students the ability to list factors in an organized way.

**Example:** Find all the factors for the number 12 using the Compression Method:

**Step 1:** Write the number 12 with brackets. 12: {        }

**Step 2:** Write the *identity factor pair* for the number 12: 12: {1,        , 12}

$\rightarrow \leftarrow$

**Note:** The only possible factors left that are between 2 and 11, inclusive.

**Step 3:** Is the number 12 even? Yes, divisible by 2. 12: {1, 2,        6, 12}

$\rightarrow \leftarrow$

**Note:** The only possible factors left that are between 3 and 5, inclusive.

**Step 4:** Is the number 12 divisible by 3? Yes. Check, 1 + 2 is 3. 12: {1, 2, 3, 4, 6, 12}

$\rightarrow \leftarrow$

**Note:** There are no whole numbers between 3 and 4. Done!

**Note:** It is highly beneficial to instruct students of the **divisibility rules** of the following numbers: 2, 3, 5, 6, 9 & 10 ; since, they are easy to remember and apply.

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**70. Place Value – Decimal Word Form.** In general, students should be able to write a decimal to the hundredths place in grade 4. The decimal point means “and,” so students must include that word in their description as well as a hyphen between two-digit numbers. **Note:** Students can always use the ‘1’ under the decimal point and add zeros directly under the decimal digits to know the place value for tenths, hundredths or thousandths (fifth grade). See guideline 60.) above for a more detailed description of the process.

Examples:      **0.03** = three hundredths.  
                      **0.4** = four tenths  
                      **6.5** = six *and* five tenths  
                      **45.93** = forty-five *and* ninety-three hundredths

**71. Adding and Subtracting Decimals.** Students have always lined-up the decimal points when they have add or subtract, but they are generally unaware of it. For example:  $5 + 24 = 5.0 + 24.0$  – there is always an implicit decimal point to the right of the one’s whole number place. However, what may be confusing to students is when the decimals are provided. Students must know to line-up the decimal points to preserve place value and add zeros if necessary to make the operation more manageable. A common mistake made by students is attempting to calculate operations horizontally. All addition and subtraction problems must be rewritten vertically to eliminate student errors. **Note:** Reinforce that there is always a decimal point to the right of every whole number (e.g.  $14 = 14.0 = 14.00 = 14.000$ )

Examples:		<div style="text-align: right;">5.00</div> <div style="border-top: 1px solid black; text-align: right;">+ 6.34</div> <div style="text-align: right;"><b>11.34</b></div>
	Add 5 + 6.34	Rewrite vertically and add zeros. Estimate to check sum: $5 + 6 = 11$ ✓
		<div style="text-align: right;">8.0</div> <div style="border-top: 1px solid black; text-align: right;">- 3.4</div> <div style="text-align: right;"><b>4.6</b></div>
	Subtract 8 - 3.4	Rewrite vertically and add zeros. Estimate to check difference: $8 - 3 = 5$ ✓ Add-up to check difference: $4.6 + 3.4 = 8.0$ ✓
		<div style="text-align: right;">9.10</div> <div style="border-top: 1px solid black; text-align: right;">- 5.67</div> <div style="text-align: right;"><b>3.43</b></div>
	Subtract 9.1 - 5.67	Rewrite vertically and add zeros. Estimate to check difference: $9 - 6 = 3$ ✓ Add-up to check: $3.43 + 5.67 = 9.1$ ✓

**Pedagogical Note:** The teacher should NOT say, “4 point 6” as the difference in the above problem. It is recommended the teacher say, “*four and six tenths*” and “*three and forty-three hundredths*” By stating the decimal name in word form, students are mentally reinforcing the place value of decimals.

**72. Ordering Decimals.** Students must be able to order decimals from greatest to least and least to greatest. If students understand magnitudes of decimals, this task is straightforward. For instance, students should be capable of equating decimals to money regardless of the place value size of the decimal. For example, if a decimal is 6.0534, then the decimal is about \$ 6.05. If the decimal is 6.54091, then its magnitude/value is about \$ 6.54. Hence, the confusion vanishes. **Note:** Students using this methodology should add zeros until all decimals are in the hundredths place value.

Examples: Order the following decimals from *greatest to least*: 0.35 ; 1.042 ; 0.4 ; 1.4 ; 0.04  
 Correct ordering:  $1.4 > 1.042 > 0.4 > 0.35 > 0.04$

## GRADE 4 - AMARA SPACED REPETITION SEQUENCING RECOMMENDATIONS

**73. Decimal Place Value – Decimal Expansion.** Students require repeated exposure until this skill is mastered. Expansion of a decimal presses students to learn and understand decimal place value and its overall value. Fourth graders need competent understanding of decimals to the hundredths place value. It is recommended that the teacher include all four forms of decimal expansion in one setting; however, the teacher should employ a build-up process, one component at a time. In the end, students are completing all four decimal expansion forms: fraction, decimal and Base 10 (both fraction & decimal).

Examples:

$$23.18 = 20 + 3 + \frac{1}{10} + \frac{8}{100} \quad (\text{e.g. fraction form})$$

$$23.18 = 20 + 3 + 0.1 + 0.08 \quad (\text{e.g. decimal form})$$

$$23.18 = (2 \times 10) + (3 \times 1) + (1 \times \frac{1}{10}) + (8 \times \frac{1}{100}) \quad (\text{e.g. Base 10 Fraction Form})$$

$$23.18 = (2 \times 10) + (3 \times 1) + (1 \times 0.1) + (8 \times 0.01) \quad (\text{e.g. Base 10 Decimal Form})$$

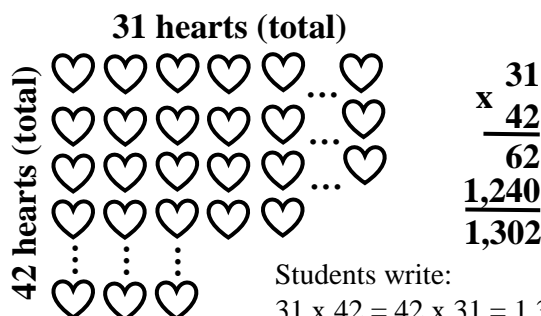
**74. Multiplication 2 digit Arrays.** In general, elementary students do not transfer well conceptually or from small number to larger number operation. They require specificity to see mathematical patterns. Thus, multiplication can be viewed conceptually as repeated addition, group models, area models or arrays. In essence, these models are all the same paradigm to an adult, but they are not to the students learning multiplication for the first time in their elementary schooling years. In array understanding, students should augment their knowledge of multiplication from small arrays to larger arrays. The teacher can show small diagrams and modified diagrams to represent any size array. It is highly recommended that the teacher state, “Find the product of the array.” In doing so, the student hears the correct vocabulary and associates that vocabulary with an operation (multiplication, in this case).

Examples:



$$\begin{array}{r} 3 \\ \times 4 \\ \hline 12 \end{array}$$

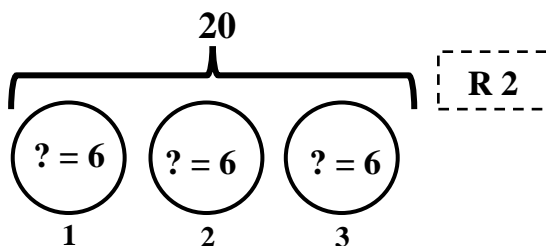
Students write:  
 $3 \times 4 = 4 \times 3 = 12$  stars



Students write:  
 $31 \times 42 = 42 \times 31 = 1,302$  hearts

**75. Estimating Products (2 x 2 digits).** Skill mastery in this task is dependent upon a student’s ability to rounding to the nearest 10 and proficiency of their basic math facts. In the spaced repetition process, the classroom teacher can quickly present two to three equations each day until mastery is demonstrated. For example,  $45 \times 82$  – estimate the product. Student answer:  $50 \times 80 = 4,000$ .

**76. Division – Whole Number – with Remainders (Revisit for Exposure).** It is highly recommended that group model diagrams be used as in problem 41.) above, so the students can visually see the mathematics in a simple diagram. The teacher should use small numbers for the same reason. Once computation skills are mastered, the teacher can provide simple word problems using a remainder for students to apply their computational skills and understand the remainder’s meaning. For instance, “20 people are attending a concert. If 6 people at most can ride in each car, will 3 cars be enough to take them to the concert?”



$$\begin{array}{r} 6 \text{ R } 2 \\ 3 \overline{) 20} \end{array}$$

20 separated into 3 equal groups (cars) yields 6 (people) in each group (car) with 2 left over. Or,  $20 \div 3 = 6 \text{ R } 2$ .

Using the diagram, **No**. 4 cars are required to transport 20 people. An extra car will be needed for the remainder (2 people). Answer: **4 cars needed, not 3 cars.**

# GRADE 4 - AMARA SPACED REPETITION SEQUENCING RECOMMENDATIONS

**77. Proper Fraction, Improper Fraction and Mixed Number Identification (Review).** As students become more accustomed and numerically fluid with fractions and mixed numbers, it is a good idea to revisit vocabulary with identifying examples. Thus, a teacher can show examples of each form and students can respond using their hands for each type. For example, students can raise their right hand, if the fraction is a proper fraction, or extend their hand forward if the teacher's example is an improper fraction. If the teacher's example is a mixed number, then the students raise both hands, simultaneously.

**Note:** A significant number of students struggle with the of a delineation fraction between an improper and proper fraction. The teacher should provide significant examples of that fraction (e.g.  $2/2$ ;  $3/3$ ;  $4/4$ ; etc.). Additionally, ALL students should readily know fractions with equal numerator and denominator (i.e. improper fractions of  $2/2$ ;  $3/3$ ;  $4/4$ ; etc.) are all equivalent to one whole or 1.

**78. Proper Fraction – Closest to Zero, One-Half, One Whole (Review).** Understanding estimations of proper fractions is another useful review. A teacher can show repeated proper fractions and students can write or signal if the fraction is closest to 0,  $\frac{1}{2}$ , or 1 Whole. **Note:** It is recommended that teachers provide proper fractions which students can mentally halve the denominator.

Examples:  $2/10$        $7/16$        $1/14$        $16/20$        $1/8$        $3/8$        $4/1,000$        $52/100$        $456/1,000$

**79. Measurement Practice** – Use the short measurement warm-ups located at the website's address located in the footer. This 3 to 5 minute measurement warm-up series includes linear, capacity and weight/mass for both customary and metric units. The students can quickly complete the measurement warm-up and flip the sheet over for additional spaced repetition skill work.

**80. Angle Identification.** Identification of the following angles is important for student success when estimating and measuring the four types of angles: acute, obtuse, right and straight angle. If the student knows these angles well qualitatively, then when measuring angles using a protractor, students possess the skill to read the correct angle measure on the protractor.

The angles that a fourth grader must know have the following definitions:

**Acute Angle** is an angle whose measure is between  $0^\circ$  and  $90^\circ$ .

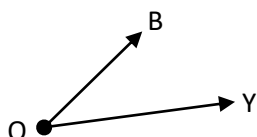
**Right Angle** is an angle whose measure is equal to  $90^\circ$ .

**Obtuse Angle** is an angle whose measure is between  $90^\circ$  and  $180^\circ$ .

**Straight Angle** is an angle whose measure is equal to  $180^\circ$ .

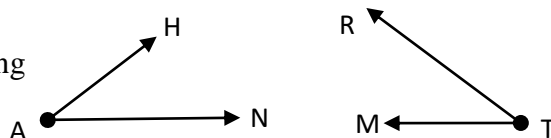
## Angle Designation:

A basic angle forms an intersection point (i.e. a vertex) between two **rays**. An angle is named by using three letters, but the **middle letter** must always be labeled as the **vertex**.



The **acute angle** shown to the left may be labeled in either two ways:  $\angle BOY$  or  $\angle YOB$ .

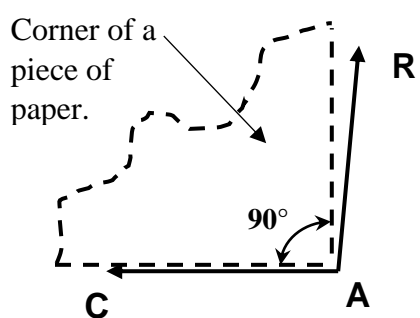
**Note:** The teacher can use two-yard sticks or meter sticks and form the four types of angles as students name the formed angle. It is highly recommended the teacher form angles with openings on the left and right, so students become accustomed to viewing angles in both directions. A protractor measures angles that open to the left and right. In doing so, students Invariably demonstrate more difficulty reading and measuring angles on a protractor like  $\angle MTR$  than  $\angle HAN$ .



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**81. Estimating Angles.** Estimating the measure of acute, right and obtuse angles is an essential skill to assist students while using a protractor. When a student recognizes an angle as either an acute or an obtuse angle based on estimation, they rarely misread the protractor's two arcs of measurement lines. It is highly recommended that students visually know the approximate measures of  $45^\circ$ ,  $90^\circ$  and a  $135^\circ$  angle. The  $90^\circ$ -degree (right) angle can also be visually taught as a corner of a piece of paper. If the piece of paper fits exactly in the angle, then the angle is a right angle with a  $90^\circ$  measure. The  $45^\circ$  angle is half of the corner of the paper (i.e.  $90^\circ \sim$  right angle). The  $135^\circ$  angle is a  $90^\circ$  degree + the  $45^\circ$  degree summed. It is highly recommended that students practice creating and approximating these three (3) angle measures using their two hands. If done so a couple times during the spaced repetition morning session, and students will own these three angle measures.

**Note:** If an angle is close to a  $90^\circ$  angle, but NOT a  $90^\circ$  angle, students can use a the right corner of a piece of paper to determine if the angle is an acute or an obtuse angle. If they fit the paper over the angle and align with the angles' bottom ray, the student can determine if the angle is an acute or obtuse angle, visually. If the student can see the angle's vertical ray, then the angle is an obtuse angle. If they canNOT see the vertical ray, it is an acute angle (i.e. meaning the  $90^\circ$ -degree corner of the paper is covering the vertical ray). See example below.



Line-up a corner of a piece of paper ( $90^\circ$  angle) inside the angle CAR. Make sure that the point of the paper's corner directly touches vertex A on angle CAR. The bottom edge of the paper must be parallel and placed directly adjacent to the ray AC. Ray AR remains visible. Hence, the angle CAR must be larger than  $90^\circ$  --- or an obtuse angle. If Ray AR cannot be seen (meaning that corner of the paper is covering it), the angle CAR must be less than  $90^\circ$  --- or an acute angle.

**82. Protractors: Measuring Angles.** Using a protractor correctly takes a lot of practice, and it should be expected that students will struggle to accurately use a protractor. Generally, students will make mistakes in three primary ways.

- 1.) Students will not center the protractor on the angle's vertex.
- 2.) Students will struggle to center the vertex on the protractor and line-up the bottom ray with the protractor's  $0^\circ$  or  $180^\circ$  line. In not doing so, the students will not obtain an accurate angle measure.
- 3.) Students will not **estimate** the angle as an acute or an obtuse angle, and they will read the wrong measure from the protractor's dual arc of angle measures. For instance, if they INITIALLY estimate the angle to be an obtuse angle, then they will read the protractor's measurement for an obtuse angle and not as an acute measure.

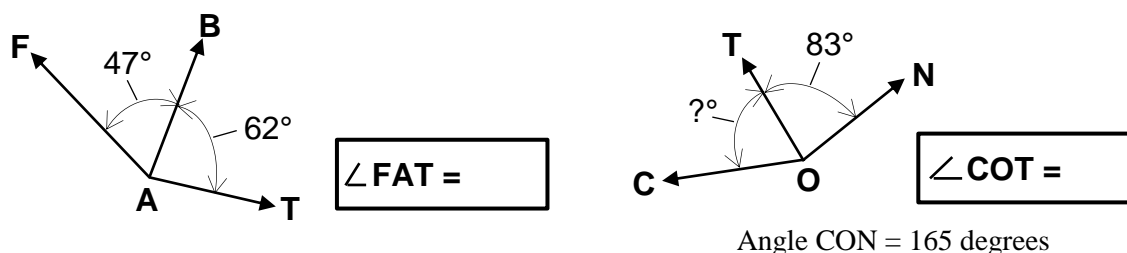
There are two ways to for students to practice measuring angles in a rapid spaced repetition session. First, the teacher displays a translucent protractor on an angle using a document camera, and the students read the angle's measure from their desks. Second, the teacher prepares a couple angles on a half-sheet of paper, and the students use a protractor to measure the angle. Both methods should be used during the spaced repetition skill mastery process.

**83. Protractors: Summing Adjacent Angles.** This skill is dependent upon the students' addition or subtraction skills and understanding angle measures. It is a conservation measurement principle and students must comprehend that they are summing two adjacent angles to obtain a cumulative angle measure. It requires very few repetitions for students to become adept and master this skill. **Note:** There

## GRADE 4 - AMARA SPACED REPETITION SEQUENCING RECOMMENDATIONS

is a subtraction possibility to this adjacent angle determination. For instance, the total combined angle measure is given, and students are required to subtract one of the two angles from the total to compute a missing adjacent angle (e.g. example shown on the right).

Examples:



### 84. Add Skills at the discretion of classroom teacher based on professional judgement and experience.

**Author's Note:** A daily spaced repetition session should require between 5 to 12 minutes of time. However, some teachers vary this time depending upon the skill and circumstances in their classrooms. It is important to note that the teacher must be highly organized in order to move quickly through the process and maintain student engagement. It is highly recommended that the teacher use a diagnostic medium in these sessions. For example, students that struggle academically should be seated in the classroom so the teacher can readily observe their work and accuracy. As those students master the skills, then the teacher knows to replace the mastered skill with a new one. The teacher can also work with those few students either individually or in a small group to ensure that they master the skill at a later time in the class or day. ***A teacher's objective must be mastery of grade level math skills by ALL students or a numeracy gap will foment and widen in later grades.***

Additionally, a couple weeks later, for instance, the teacher can spiral review to previously presented math skills and guarantee with absolute certitude --- that all students have indeed mastered grade level or prior grade level skill(s). Application mathematics in the arithmetic elementary grades are generally given in the form of a short 'story' or 'word' problem. These word problems are nothing more than a combination of discrete arithmetic skills listed in this document. If those arithmetic skills are mastered, there is a high probability that students will not be overwhelmed and easily solve arithmetic application problems correctly.

*Blaine C. Helwig*