Accelerated Math Fact Student Mastery

An effective and efficient process so elementary and middle school students can master ALL four math fact arithmetic operations (addition, subtraction, multiplication and division)

PART 3 of 3

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Executive Summary

Edwin Rommel, a World War II German general, stated in 1944 as he was inspecting the Atlantic Wall defensive positions against Allied (i.e. Canada, Britain and American) invasion along the France and the Benelux countries' coastlines, *"When one tries to defend everything, they defend nothing."* Rommel believed that the German Army should specifically prepare for an amphibious Allied landing at only two of the most likely beachheads – not ubiquitously – which diluted the overall effectiveness of their military resources and personnel. Rommel's famous statement is a testament to both value and importance in organizational structure – <u>everything does NOT possess the same priority level</u>. If any elementary school – especially a Title 1 school – is to be academically successful, the campus' administration must isolate the most important aspects of their daily work. Those few factors providing the greatest return on student learning must be identified and pressed.

As far as elementary mathematics is concerned, a structured daily numeracy program is one of those crucial instructional elements. It must be differentiated and address both aspects of numeracy – basic math facts and general processing math skills (i.e. place value, number lines, rounding, etc.). If these two numeracy areas are not directly addressed, students' problem solving ability will be significantly depressed. The school's administration and teaching faculty must value and convey to students that learning and mastering math skills – basic math facts and processing skills – possesses both extrinsic and intrinsic mathematical value and importance.

The unintended benefits of an effective stop-gap numeracy program at a Title 1 school not only afford a school heightened mathematics performance, but also yield additional positive outcomes for the school. For example, students qualifying for special education and student discipline problems are greatly reduced due to the fact that students are accelerated 'back' to grade level. Students' math skill gaps are greatly reduced and students do not 'appear' to possess learning issues. On-grade level students also actively and appropriately engage in the daily lesson, significantly reducing student disruptions. Simply put, administrators and teachers must understand and communicate to students that mastery of arithmetic numeracy for both math facts and math processing skills, and this instructional focus must be a school-wide primary and intermediate grade level daily press.

It is less difficult for a student to learn his or her numeracy process math skills than their four (4) math fact operations. Historically, an age-old battle for classroom teachers has existed to develop an effective system and the energy to engage young students to master their math facts. However, since the advent of the digital computer, this process is not nearly as difficult as it was in the past. A standard computer math fact program will be highly effective with a minimal student group, but that is not the goal of any school. The school's primary objective of total students mastering their math facts should produce results between 98 and 100%. To accomplish that level of success – the computer, administrator, teacher and an effective pedagogical system must be soundly developed and implemented.

With the use of Formative Loop (Numeracy) as a digital conduit of a five (5) minute daily paper pencil assessment, this entire process can be standardized, school-wide. The program affords efficient tracking, standardization and the ability to control the process for all classrooms at a school – ending islands of 'classroom' academic excellence at a Title 1 campus to one where all classrooms demonstrate high student performance. Again, the administration and classroom teachers play a vital role in this process. A computer alone is not going to educationally 'save' a child – that is the primary role of campus educators. Educators must diagnostically evaluate and monitor a student's daily work while facilitating and motivating children to academic prowess

In this white paper, pedagogical methodology is described that combines digital and human elements in a parsed process that results in nearly all third (3rd) grade students mastering all four math operations. The procedure relies on a methodology that prepares children for success at the next math fact operation with very short and effective classroom mini-lessons (e.g. spaced repetition) in conjunction with the stair-stepped Formative Loop numeracy math fact procedure. The entire process is sequentially outlined in the white paper. Success only requires implementation and the marshalling of school personnel in a coordinated effort.

This numeracy formula has helped produce two (2) urban Title 1 National Blue Ribbon Schools, multiple Gold Ribbon Schools (At-Risk) and a myriad of Texas Education Agency (TEA) high academic performing Title 1 campuses – with economically disadvantaged student enrollment above 90 percent. Finally, it is important to note that all elementary mathematics is a one-year academic turnaround, regardless if the school is a Title 1 school or not. Heightened mathematics performance affords additional benefits in both the literacy and science instructional focus. A one page white paper thoroughly explaining this situation is available for free download at the website address referenced in the footer located in the 'Expertise' tab.

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Qualifying Statement

This white paper is 14 pages in length. However, it is intended as a stand-alone paper, so the reader/educator understands the outlined process specifically at every juncture in the pedagogical process at their schools. However, this document is also a companion white paper to "90 Minute Math Block – Putting All the Pieces Together." It is part 3 of 3 of that series intended to thoroughly expound upon improving elementary and middle school mathematics performance.

The methodology has taken almost 15 years to completely develop into a replicable, pragmatic procedural philosophy. Hence, for every one page that the educator reads in this document, facetiously, it took approximately one year of thinking and consideration to simplify the process. In retrospect, I would rather have read this paper in 30 to 45 minutes than engage 15 years of my life working in the public schools to develop it.

It is absolutely essential that students master their basic four (4) operation math facts in elementary school to provide a command of algorithm operations and relative ease in problem solving. There are so many areas where math fact mastery benefits other math skill operations; however, if the students enter an algebraic or geometric environment without math fact skills and other dependent arithmetic skills at high efficacy levels, their algebraic or geometric understanding will be negatively impacted.

If an educator has any questions or suggestions to improve this process, please contact us via the website address in the footer. It is the overarching hope and objective that children become more adept mathematics students. If your comments or questions facilitate this learning process, any and all constructive criticism is highly appreciative.

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Accelerated Math Fact Student Mastery

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Accelerated Math Fact Student Mastery

One of the most difficult pedagogical problems faced by elementary mathematics teachers is achieving the seemingly impossible: student math fact mastery. These four operations (addition, subtraction, multiplication and division) have been the bane of the vast majority of educators' existence from their first day as a teacher until the day they retired. There is no magical effortless solution, but there is a process that will make it much easier. However, like all valued human endeavors, the process requires effort from both the classroom teacher and his or her students. This white paper proposes a true and tested procedural methodology that makes this process much more viable. With the use of the Formative Loop Numeracy Program, the monitoring and tracking of student fact progress through a five minute daily assessment is extremely efficient.

Using the combination of the Formative Loop digital conduit and effective classroom pedagogy, the teacher is preparing their students for success on the next math fact assessment as they progress and master their four (4) math fact operations. For instance, as students are mastering their addition facts, they are systematically prepared for success in both their subtraction and multiplication facts.

Section 1

Value and Importance of Student Math Fact Mastery

The first question that must be addressed with regard to the value and importance of student math fact mastery is the following: Is student math fact mastery worthy of the necessary time investment by campus educators? The answer to this question may be addressed below by both State and National Mathematics Standards and influential mathematics organizations and advisory panels that have researched this topic.

1.) A student's mathematics ability is highly dependent upon a student's ability to fluidly work with basic number operations. A highly numerate student demonstrates problem solving prowess which is directly related to their basic skill level. Hence, the Texas Essential Knowledge and Skills (TEKS) and the Common Core (CC) standards both require mastery of all four math fact operations in elementary school years. As an example, the 2010 Common Core Standards clearly state:

   a.) Standard 2.OA.2. Fluently add and subtract within 20 using mental math strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

   b.) Standard 3.OA.7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division. By the end of Grade 3, know from memory all products of two one-digit numbers.

   c.) Similar TEKS requirements to the two Common Core Standards named above are also stated in the State of Texas Mathematics Standards.

2.) Many mandated State assessments using Common Core Standards and the Texas Essential Knowledge and Skills are currently timed. For example, in Texas, the STAAR examination for elementary schools must be completed by students in 4 hours. Students must be proficient numerically and fundamentally in math facts and numeracy skills to

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have the capability to complete each problem efficiently in that time limit. Hence, the following three questions are taken directly from the Fifth (5^{th}) grade, 2016 State of Texas Mathematics STAAR (State of Texas Assessments of Academic Readiness) spring assessment:

a.) Example Problem 1: Scott drank 3.5 bottles of water yesterday. Each bottle contained 1.2 pints of water.
What was the number of pints of water Scott drank yesterday?

b.) Example Problem 2: Phoebe divided her rectangular vegetable garden into three sections, as shown in the interconnected (not dimensionally labeled) drawing.
The potato section is a square with a side length of 7 meters. The carrot section is a square with a side length of 5 meters.
What is the area, in square meters, of the corn section of Phoebe's garden?

c.) Example Problem 3: At a clothing store, Zoey bought 2 shirts for $7.25 each and 2 pairs of jeans for $24 each. She used a coupon for $10 off the total price of the clothes. The discounted price of the clothes Zoey bought can be found using the following expression: [2(7.25) + 2(24)] - 10
What is the discounted price in dollars and cents of the clothes Zoey bought?

3.) The 2008 final report on the National Mathematics Advisory Panel's principal message cited, "Use should be made of what is clearly known from rigorous research about how children learn, especially recognizing…the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic…quick and effortless…recall of basic math facts." Additionally, the report cited, "Computational proficiency with whole number operations is dependent upon sufficient and appropriate practice to develop automatic recall of addition and related subtraction facts, and of multiplication and related division facts."

4.) From the National Council – of Teachers – of Mathematics (NCTM) for Grade 4 (a similar statement for grade 2 addition and subtraction is also cited in their cannon) cited the following canon illustrating the importance of math fact mastery for elementary students:

Number and Operations and Algebra: Developing quick recall of multiplication of multiplication facts and related division facts and fluency with whole number multiplication.

Students use understandings of multiplications to develop quick recall of the basic multiplication facts and related division facts. They apply their understanding of models for multiplication (i.e. equal-sized groups, arrays, area models, equal intervals on a number line), place value and properties of operations (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalized methods to multiply multi-digit whole numbers.

The importance of elementary students mastering their operational math facts is a noted requirement from both Texas and Common Core Standards. The value of math fact mastery is also highlighted by several of the most recognized mathematics organizations with regard to public education and pedagogy in the United States. The mere fact that math fact mastery is a required mathematics skill does not deter classroom teachers' pedagogical difficulty in pressing students to successfully mastering math fact automaticity.

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However, using the Formative Loop Numeracy Program in collaboration with a consistent and structured methodology significantly reduces the lack of success for students in the general process.

Section 2

Teacher and Administrator Math Fact Support Role – Of Vital Importance

It is absolutely imperative that both the teacher and the administration support and press students throughout the math fact learning process. If this step is not actively incorporated, student performance is impacted accordingly. With regard to the five (5) minute assessment, teachers must NOT simply distribute the practice or assessment math fact sheets with an air of program obligation – as if students will be self-motivated to excel based on their own intrinsic motivation. Unfortunately, this type of classroom climate and culture encourages elementary-aged students to an attitude and resignation of mediocre academic indifference. Of course, elementary and middle school students do not possess long-term perspective on life. They are too young and inexperienced to realize that NOT learning their math facts or correct grammar or general academic skills will negatively impact their lives in the not so distant future. But, educators have years of experience and more than realize the impact of a poor formative education. A teacher is not working with small machines, but with young children where strong relational bonds are developed based on positive interactions. An educator not only imparts and facilitates knowledge and learning, but they are an influential guide and a motivating academic and social coach. A computer is not going to provide students with a fundamental and rigorous education; however, an adult that is motivating, encouraging and pressing elementary students to higher levels of personal success will.

Over 500 years ago, William Shakespeare wrote the metaphoric lines, "All the World's a Stage." The classroom and the school IS the teacher's stage. In short, teachers and campus administrators must promote and convey value and the importance of student learning in an animated and relational way to motivate their students to academic and social heights. Educators must openly demonstrate value in their daily pedagogy and skill lessons via encouragement, guidance and celebrations of success for all students.

Section 3

Formative Loop Numeracy Program – Pragmatic Use and Understanding

As many campus educators implement a school-wide math fact mastery program, there is an initial reaction of surprise and disappointment. Classroom teachers and administrators discover the vast majority of their intermediate elementary students never mastered their addition and subtraction facts – a primary grade skill, let alone their multiplication and division math facts. The realization of this stark math fact reality offers six (6) clear curricular concerns for the school. First, an accountable primary grade program in this mathematical realm is needed immediately. Second, their students are generally unprepared for grade level mathematical problem solving rigor. Third, daily consistent and accountable general numeracy skill program and math fact program is overdue for the intermediate grade levels. Fourth, students' math fact and process skill progress should be handled digitally to facilitate time efficiency for each classroom teacher. Fifth, other Title 1 schools with the same challenging student demographics and mobility have successfully 'front-end loaded' this numeracy program and instructional routines at their campuses and have performed at similar academic levels as non-Title 1 schools. Finally, this school year's implementation and work will reap tremendous benefits in succeeding school years.

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The basic workings of the Formative Loop Numeracy Program is a differentiated math fact and process skills numeracy program that affords teachers and administrators the following rudimentary functionality:

- The ability to monitor and track every student in a differentiated skill in every classroom from 1st through 8th grade in both math skills development and math fact operation. Administrators can digitally monitor each classroom and grade level at the campus using real time data. The administrator can also standardize and readily evaluate all student skill progress and mastery at a campus independent of the student enrollment – large or small.
- The capability to accelerate students 'back to grade level' who are month(s) or year(s) behind academically due to built-in previous grade level skill parallel sequencing. The numeracy program is differentiated – meaning that every student in the classroom may be on a different skill level, but their progress and skill organization skill sheets are automatically printed and tracked digitally. This affords a highly organized structure with an efficient teacher-friendly daily routine.
- The functionality to implement a daily 5-minute paper-pencil assessment or a separate sequencing using only math facts and/or only math processing skills. Hence, an intermediate classroom teacher has the option of running two (2) - 5 minute assessments each day – one for only math facts (i.e. addition, subtraction, multiplication and division), and one daily assessment for math processing skills (i.e. place value, rounding, number lines, etc.). It is highly recommended to implement a dual system – especially in 3rd grade, and move newly enrolled students to the school in a dual assessment system for 4th and 5th grades, so students do not get mired in math facts to the detriment of skill development in math processing skills.
- All four math fact operations are sequenced into individual assessments of 1's, 2's, 3's, 4's, 5's, 6's, 7's, 8's, 9's and a final mixed assessment. There are twenty-five (25) one-digit math fact computations for 1st graders, fifty (50) one digit computations problems for 2nd graders and one hundred (100) computation problems for students beginning in 3rd grade Each student is given a daily five (5) minute timed math fact exercise (or process math skill). The assessment is checked and inputted into the Formative Loop Numeracy Program as either a Pass (P) or Fail (F) outcome. If the student 'passes' the student is moved to the next sequenced exercise. If not, the student is assisted by the teacher, volunteer, other school personnel or administrator, and that student is reassessed the following day on another version of the same math skill.
- The numeracy program is designed so that if the student is unable to pass the mixed addition 5-minute assessment, the student works through a 5-minute parsed assessment of 1's, 2's, 3's, etc. each day until they are successfully prepared to pass the final 5 minute mixed assessment of each math fact.
- Human grading is required – for diagnostic reasons: Teachers or support personnel may review only the 1st, 4th, 7th, and 10th rows on the stepped-up (1's, 2's, 3's, etc.) math fact assessments. However, on the mixed assessment, ALL 100 problems should be checked for correctness. On the final mixed assessment, the teacher has several options if the student is NOT completing the exercise.
    1. If the student is showing signs of progress – more math facts each day correct, it is recommended to allow them to continue each day with nightly practice. They will master the sheet within an expected number of school days. The student should complete their unfinished assessment for that school day in addition to the practice sheet for nightly homework.
    2. **The quick five (5) minute daily assessments are a measure of Verification of Mastery (VOM) of a math fact or a general processing skill**. On each skill sheet

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answer key, there is a 'black star' for a recommended mark that demonstrates skill mastery; however, the classroom teacher may opt for a different metric on each daily assessment depending on a student's specific situation (e.g. 504 students or students receiving special education services – refer to the student IEP – Individual Education Plan). The standard or general passing percentage for regular education intermediate students on math fact assessments is 95/100 correct without skipped fact problems.

3. The grader/teacher is readily able to identify which specific math facts are difficult for the child. It is recommended that the teacher/grader/volunteer focus only on the few problems the student is NOT correctly answering. The three methods below (a., b. & c.) plus the note expatiate on instructional options:
    a. The students should be directed to turn their sheet over after the daily assessment and write those specific problems 5 to 10 times each that they are not answering correctly.
    b. The teacher can create handmade flash cards using blank 3 inch x 5 inch index cards of ONLY the few missed math fact problems.
    c. If the student is struggling, place them back on the 2's, 3's or 4's, etc., the student may need to work through the process, again. This occurs most often if the student has been 'cheating' by completing the first row of the assessment and copying those answers on each of the row beneath. Hence, the student did not learn the math facts to memory.
    d. ***Important Note:*** If the student is missing the math concept on either the math fact skill or on the math processing skill, the student is in dire need of a direct intervention. Otherwise, the student will continue to attempt the skill unsuccessfully in upcoming assessments.

- The Formative Loop Numeracy Program contains a 'Resource Library.' The Library consists of a 'General' Section and a further breakdown of related skill sheets and resources for grade levels from 1st grade through 8th grade. In this Library, Math Fact practice sheets in all four operations are parsed in 1's through 10's and final mixed assessments. The Library also has skill practice sheets for "Multiples" – numbers 1 through 12 (available in quarter, half or full practice sheets), "Making 10" and "Finding the Missing Factor." There are over a thousand additional resources in the Library – including flashcards for all four math operations. All practice skill sheets and pedagogy papers are available for immediate free download.
- ***DAILY*** – The teacher should provide each intermediate (i.e. 3rd grader and above) student the mixed – final assessment – addition assessment using Formative Loop (100 addition mixed – 1-digit addition problems completed in 5 minutes). If a student can score at least a 65 to 70 percent score without skipping problems and without practice, they know them. The student only requires a little more practice to demonstrate mastery of their addition math facts. Hence, it is recommended the student remain on the mixed final addition assessment and within two to four days the student will correctly complete 95 out of 100 problems.

  However, if the student is scoring far less than 60 percent of the mixed assessment (without practice), that student should be moved down immediately to the structured sequencing of completing the 100 addition facts of the 1's only assessment, 2's only assessment, 3's only assessment, etc. Please note: Students who have demonstrated a mastery knowledge of 1's, and 2's,

should be skipped and placed in the program to the 3's position to eliminate a loss of unnecessary days of student practice on facts that they already possess mastery.

Primary-aged students should generally begin on the stepped-up math fact sheets (1's, 2's, 3's, etc.). However, if a first or second grader is capable of demonstrating proficiency on the final assessment, the student should be provided that assessment level.

Final Five (5) Pragmatic Notes:

1. Intermediate grade level students: Attempting the 1's only assessment, 2's only assessment, etc. are also given five (5) minutes to complete the daily learning opportunity. *The student should use a separate piece of paper* to cover the horizontal row(s) completed so they do not copy the previous line of addition facts by rote. *Students should also complete each row from left to right,* so they memorize the fact and not just complete the row by skipping around using a multiple strategy. **The objective of the stepped-up Formative Loop approach is to slowly and deliberately build the student's knowledge discretely to complete a mixed final assessment.**
2. Students should be monitored during the five (5) minute assessment by the classroom teacher to ensure that each row of facts is approached as a 'new' row of addition facts. As previously stated, using this slow process of building at each level of 1's, 2's, 3's, etc., students easily complete the final mixed addition fact assessment after completing the addition 9's. After completing the addition mixed, the student is automatically transitioned to the subtraction mixed, and the process repeats for that math fact series.
3. When a teacher/grader is hand-grading each assessment, they can analyze student work for common errors that are easily rectified with a quick intervention. However, if a student's paper is wholly incorrect and notably prior grade level skill deficient, it is recommended the teacher evaluate the paper for a commonality of the student's errors. If the student is consistently incorrect with a similar error, it is highly likely the student is only behind academically and NOT a student in need of a special education diagnostic assessment. However, in a similar situation, if the student's errors are consistently random and nonsensical day after day, further diagnostic services may be necessary.
4. In the Formative Loop Resource Library – General Section – there are several short one-page papers that expatiate and expound on helpful practical 'tips' in using this numeracy program.
5. Finally, it is also recommended that students are given numeracy homework each night. This extra skill practice sheet is selected (i.e. option checked) on the Formative Loop Program and easily printed on a school copy machine as is the next day's five (5) minute assessment.

Section 4

Math Fact Performance Celebration Awards – Formative and Summative Incentives

As a former classroom teacher and campus principal, I firmly believe in the power of incentives to encourage and motivate students. As expected, students respond differently to one incentive over another. Some students desire to succeed and perform is so internally driven that they require no external incentive, whatsoever. Hence, effective formative and summative incentives often depend on each student's personality; however, there are general incentives that always seem to have a large-scale positive impact.

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A school-wide based incentive system or an individual classroom teacher can set-up an incentive for both formative and summative math fact student performances. In my experience, the following are possible incentives that have motivated large numbers of students mastering their math facts:

1. The Formative Loop Numeracy Program has embedded a customized 'Certificate of Achievement' acknowledgement that automatically prints each time a student successfully passes each of the four math fact operations. Surprisingly, this simple incentive is highly effective with both elementary and middle school-aged students.
2. Primary teachers can use small inexpensive stickers or other appropriate incentives as they choose. It has been my experience that the vast majority of first and second graders are highly motivated by stickers as they successfully complete their addition and subtraction fact requirements at their developmental level. Again, the educator's energy and approach to the celebration of student success is absolutely key with students of any age.
3. On students' daily five (5) minute assessments, writing encouraging notes is highly motivating to students of all ages. For example, a simple statement is so meaningful to a child: good work, keep at it, excellent, outstanding, wow, congratulations, well done, you did it, etc. The child realizes that their diligent efforts are noticed and recognized.
4. It has been my experience that the most effective summative incentive for a student completing all four math fact operations is the "Texas Math Fact Drivers' License." As a principal and classroom teacher, I have never found one student that did not appreciate their personalized Math Fact Drivers' License. Both the instructions to make these Texas Drivers' Licenses and MS Word templates may be downloaded for free at www.thenew3rseducationconsulting.com under the "Expertise" section of the website. When a child earns their Math Fact Drivers' License, a short recognition in the class with students' applauding the recipient's efforts is a meaningful manner to create and establish a recognized classroom culture of academics. Another noteworthy venue is a weekly school assembly for student math fact recognitions. Note: If a school or teacher would like this incentive for their use in a State other than Texas, please use the contact information on 'the New 3Rs' website to request this free alteration. The template is easily modified for any state other than Texas by our Photoshop software.

Section 5

ADDITION FACTS – Classroom Specifics and Pedagogical Understanding

As mentioned in Section 3, intermediate grade level students are given a Formative Loop daily math fact assessment of one hundred (100) single digit addition facts in five (5) minutes. In order to demonstrate numerate mastery, the student must score a minimum of at least ninety-five (95) percent correct (without skipping any addition one-digit problems). The mastery assessment standard was designed based on the following criterion: Each addition math fact problem should be successfully completed in approximately 3 seconds. Since there are a hundred (100) single digit addition problems per assessment, there exists a sufficient number of every one (1) digit addition combination from 0 to 9 (i.e. 2 + 3; 5 + 6; 9 + 7; 0 + 4; etc.) to demonstrate student competency for all sums.

Memorization and recall of addition facts is aided by the commutative property of addition. The commutative property of addition states that "a + b = b + a" or "4 + 5 = 5 + 4". This dissects the memorization by approximately half of the total number of one-digit addition facts. The vast majority of

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students will not struggle with the automatic recall of one-digit addition problems containing a 'zero' (e.g. 4 + 0) or the addition of a 1 to any given one-digit number (e.g. 1 + 7). Hence, the total number of single digit addition math facts to memorize with the 2's, 3's, 4's and 5's is a total of 26 discrete facts. Due to the commutative property of addition, there are only 10 more facts to memorize with regard to the 6's, 7's, 8's and 9's. In total, there are only 36 discrete addition facts that must be memorized for students to master their addition facts.

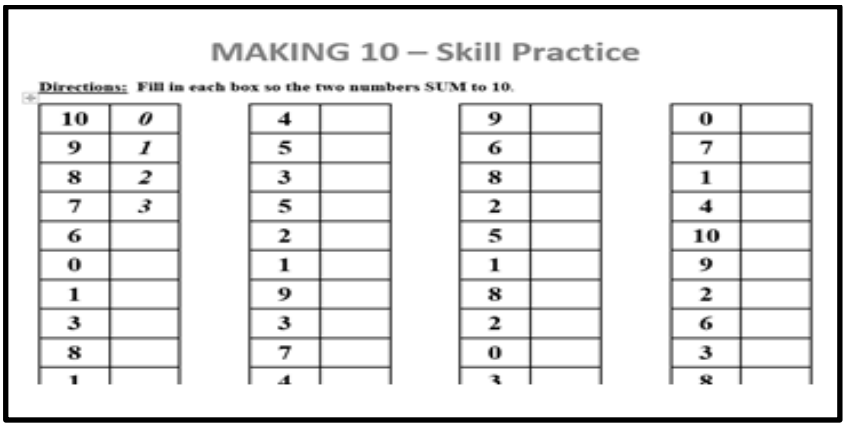
The following pedagogical procedure is recommended for intermediate grade (3^{rd} – 6^{th}) level students to facilitate the mastery of their addition facts:

a.) An effective and time-tested classroom technique is requiring students to memorize the addition doubles. Hence, 3 + 3, 4 + 4, 5 + 5, etc. The teacher can logically demonstrate that if 6 + 6 equals 12, then 5 + 6 = 11 and 6 + 7 = 13. Many students can understand the addition of the one less and one more on doubles as long as the teacher consistently practices in short spurts each instructional day with either whole class instruction or a small group of students.

b.) Extra skill practice sheets are readily available for download in the Formative Loop Resource Library 'General Section'. Each student may be given short practice sessions with regard to their progress level (2's through 9's or final mixed assessment) during lesson transition times.

c.) The students should be pressed and held accountable. The students will slowly but surely progress with practice and encouragement. As expected, students and adults both enjoy to be complimented when they are doing well at some given task. As mentioned in Section 4, an extrinsic student reward system can be established school-wide or by each individual classroom teacher.

d.) The teacher should also provide quick mini lessons in students 'Making 10.' This skill will be used when students master their subtraction facts as they complete their addition fact final assessment. Students should also be given five (5) minutes to complete the Making 10 Skill Practice Sheet. Classroom teachers can practice tactilely with their students as the teacher holds up four (4) fingers, students quietly respond with (6) fingers. Students quickly master this skill. Again, the teacher is preparing their students with needed support skills in advance of the subtraction and multiplication math fact skill assessments. This practice sheet shown in Figure 1 below is available for immediate download in the Formative Loop Resource Library – General Section.


MAKING 10 – Skill Practice

Directions: Fill in each box so the two numbers SUM to 10.

10	0	4		9		0	
9	1	5		6		7	
8	2	3		8		1	
7	3	5		2		4	
6		2		5		10	
0		1		1		9	
1		9		8		2	
3		3		2		6	
8		7		0		3	
1		4		3		8	

Figure 1: 'Making 10' Student Practice Sheet

e.) After students finish their 5 minute daily math fact learning opportunity, the teacher should instruct students to turn their paper over to the blank side of the paper. The classroom teacher should instruct students to count by multiples (skip count) of a stated number. It is recommended to sequence the process in the following order: 2's, 10's, 5's, 3's, 4's, 6's, etc. The teacher may need to help students skip count slowly by 12's by moving in small segments of 0, 12, 24. Then, add a number each time in the pattern (0, 12, 24, 36), and then (0, 12, 24, 36, 48), and so forth. With a structured learning pattern the students will rapidly improve. It is recommended that the teacher require students to practice these half or quarter sheets during the instructional day as quick practice sessions for 5 minutes as the teacher prepares and transitions for their next core lesson. Nightly homework is also recommended until multiples mastery is achieved. This skill set has intrinsic value within its own numeracy level; however, it will greatly assist students in their multiplication fact mastery in the coming weeks as they learn that multiplication may be treated as 'repetitive adding.' Teachers are preparing students for the needed skill sets prior to their advancement to the multiplication math fact assessments. Students should be able to complete a Multiples practice sheet for numbers 1 through 12 in third grade in five (5) minutes. Practice sheet is shown in Figure 2 below and is available for download in the Formative Loop Resource Library – General Section.

Multiples Skill Practice

Directions: Fill in the table with the correct multiples by skip counting downward.

1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0						
1	2	3	4								
2	4	6									
3	6										
4											
5											
6											

Figure 2: Multiples Student Practice Sheet

f.) It is highly recommended the classroom teacher work with all students that begin to fall behind in the addition (or subtraction, multiplication or division) math fact process by providing extra practice. **However, if the classroom is NOT using the two separate skill sequencings available with Formative Loop (i.e. a math fact and a general math skill), the teacher can create a separate unique math fact class, and she/he can place these struggling math fact students in that section. Then, the teacher can skip the addition facts and place them on the first skill level in the next section. Consequently, students do not become mired in math facts for a significant portion of the school year.** The class can be called, for example, "*Teacher Name* Math Facts" and the entire class continues progressing with math process skills, and those few students remain accountable for learning and mastering math facts for addition (or subtraction, multiplication or division). Note: If using Formative Loop and the dual or two sections are not listed, request their use with the Formative Loop founders for that sequencing. There is not an additional cost associated with the dual streaming option, but it does require those students to complete two (2) five (5) minute daily assessments.

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g.) As mentioned briefly in Section 3, subsection 3 (b), a common pedagogical mistake often occurs when a student is taking his or her final mixed assessment in addition (or subtraction, multiplication or division). The child skips 5 to 8 of the same fact problems (i.e. 7 + 8, 6 + 7, 9 + 5 and 6 + 8) because they do not readily know them. In this case, the teacher should not provide the student a set of flash cards with ALL the addition facts. The child needs to be guided to diagnostically focus ONLY on the problems that they have NOT mastered. Hence, the teacher can quickly write out 5 to 8 index cards with ONLY the facts that the child has not mastered with the sum/difference/product/quotient on the back of the index card. That way, the child studies only the math facts that they do not know, and the success of this approach the following day will be more than apparent. If the teacher also relays to the student at the end of the school day, "I want you to study only these cards tonight. Ask your mother, father or sibling to quiz you after you have studied. I will be able to see your progress tomorrow morning on those few problems."

Section 6

SUBTRACTION FACTS – Classroom Specifics and Pedagogical Understanding

Generally, subtraction facts are the most difficult for elementary-aged students to master of the four math fact operations. Students learn single-digit addition and multiplication facts more quickly since both operations adhere to the commutative property (i.e. 2 + 3 = 3 + 2 and 4 x 5 = 5 x 4). As explained above, if the addition or multiplication of 0's (i.e. 0 + 1 or 0 x 3) and the1's (i.e. 1 + 3 or 4 x 1) are not counted, there are only 36 discrete single-digit addition and multiplication facts to memorize. These facts are easy to memorize with a little bit of work.

However, this isn't the case with subtraction facts. 8 – 5 = 3 and 8 – 3 = 5 appear as two separate subtraction facts, and quite frankly, they are two separate math facts to a young child. It may be the same fact family, but students must memorize both of these subtraction facts, since there's no commutative property of subtraction. To complicate matters, students must also memorize double-digit facts subtracting a single digit (i.e. 13 – 8 = 5 and 13 – 5 = 8). These differences are still more difficult to compute.

There is a method to increase this subtraction fact memorization process. Students will still need to memorize single-digit from single-digit subtraction facts (i.e. 5 – 3 = 2 and 9 – 6 = 3), but those facts are much easier than the memorization of single-digit numbers subtracted from a double-digit number (i.e. 17 – 9 = 8). In the latter case, teachers are able to take advantage of the previously mastered addition facts and base 10 mathematics. In short, this subtraction technique relies heavily on student proficiency of the addition of two single-digit numbers – a skill that has been previously mastered or the student would not be working on his or her subtraction facts.

While learning their addition facts, the students should have mastered the numeracy skill for "Making 10." The probability is extremely high that a student has not memorized his or her subtraction facts previously if the student needed to progress through the Formative Loop, stepped-up version of their addition facts. Consequently, it is recommended to save time and skip the student from the 'Subtraction Mixed' assessment and directly place the student on the Formative Loop subtraction stepped-up sequencing of 1's, 2's, 3's, etc.

The method is illustrated in Figure 3 on page 11 using the math fact 15 – 7 = 8 in a typical subtraction fact format.

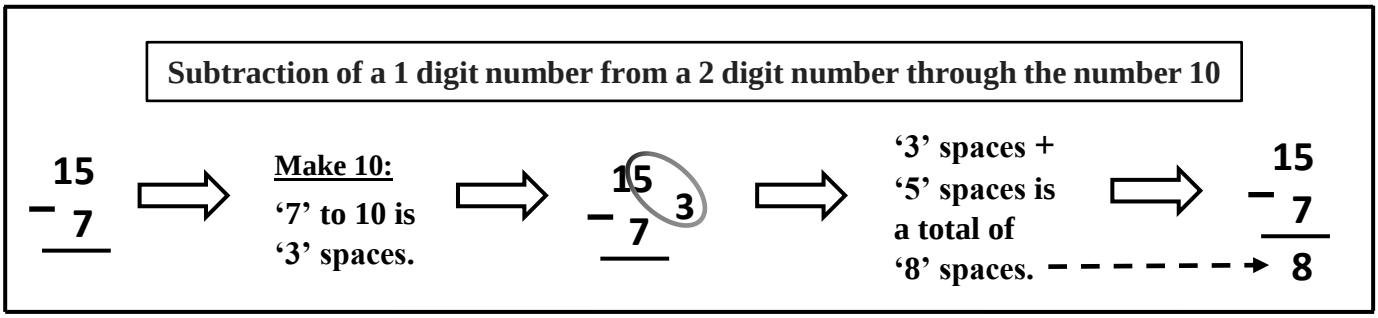

Figure 3: Physical Meaning of Subtraction Via 'Making 10'

If the student demonstrates no difficulty memorizing their single digit from double digit subtraction facts, it is recommended that the method shown in Figure 3 above NOT be used. If, however, the student does struggle with the subtraction of single digit from double digit problems, then the procedure below utilizing the previously learned addition facts and 'Making 10' practice should be considered. It is important that both of these skills should be learned prior to teaching this method, so students are not learning Making 10 and addition facts in the process. Students' subtracting computing speed will be slowed greatly if there is not automaticity with those two previously learned skills. This method is illustrated in Figure 3 above.

Students must understand the physical process of this method. Teachers should illustrate this simple subtraction math fact process on a number line. Then, students can visually understand the mathematical reasoning why this subtraction process **ONLY** is valid for 2-digit numbers subtracting 1-digit numbers – that pass through the number 10. The number line in figure 4 below clarifies the physical meaning of the process using the subtraction math fact problem (15 – 7) and its difference of 8 total spaces.

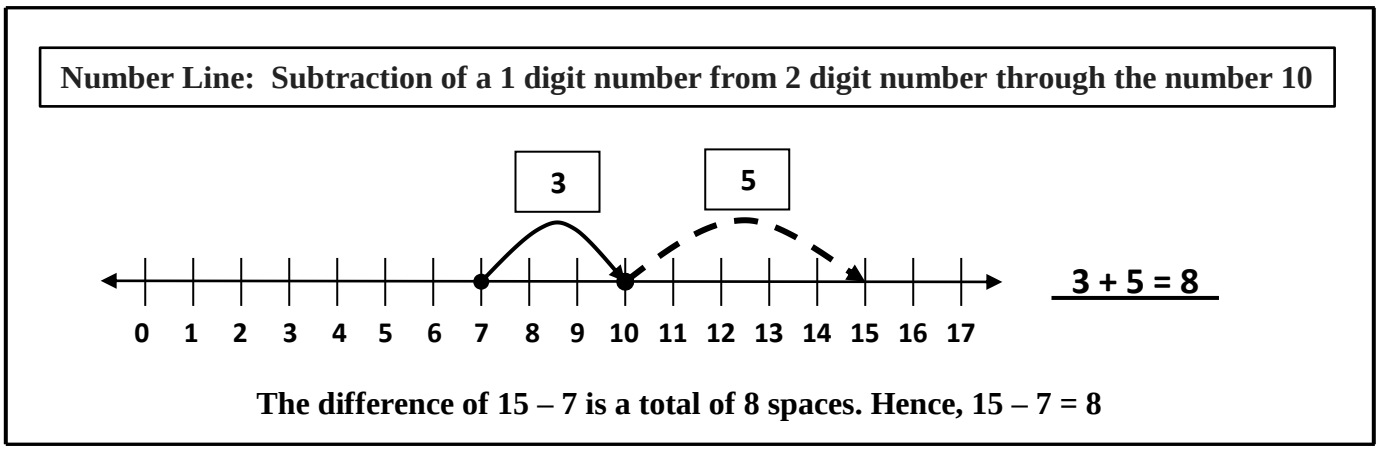

Figure 4: Subtraction Via 'Making 10' – Physical Meaning

Without using this method or some other effective process, students often becomes bogged down in their subtraction facts losing weeks of valuable learning time. This is a major reason academically successful Title 1 schools use this subtraction method and double stream Formative Loop Numeracy – implement a math fact plus a math processing skill sequence using Formative Loop in third grade. Finally, the multiples practice described in Section 6 – paragraph d – should continue until mastery. It is also recommended that homework be sent home for additional practice to any students struggling with completing this numeracy skill in five (5) minutes – a common standard for mastery level.

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Section 7

MULTIPLICATION FACTS – Classroom Specifics and Pedagogical Understanding

As with memorization of addition facts, multiplication fact mastery is aided by the commutative property of multiplication. Hence, since 4 x 5 = 5 x 4, as with addition, there are only 36 discrete multiplication facts for students to learn if the 0's and 1's are not included. As expected, students readily memorize their multiplication facts quickly with focused practice.

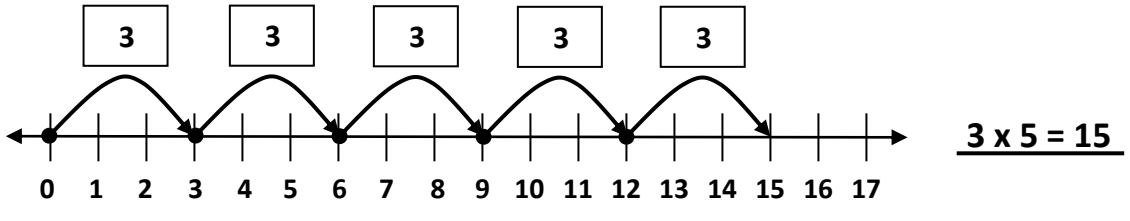
During the addition and subtraction stages of math fact memorization, the classroom teacher has previously required students to master their multiples (e.g. skip counting) for the numbers 1 through 12. This numeracy skill is intrinsically valuable in preparing students for immediate success in multiplication fact automaticity. Without realizing it, students have acquired the physical meaning of multiplication using a repetitive addition model. For example, students understand that 5 x 3 means either 3 equal jumps of 5 or 5 equal jumps of 3 to produce a product of 15. This method eliminates rote memorization of multiplication facts without conceptual understanding. The classroom teacher can further demonstrate the physical meaning by using the number line model depicted in figure 5 below.

There are two additional fundamental multiplication models that students require physical understanding – the group and area model. For example, the group multiplication model – means there is a fixed number of equal groups with an equal number of objects in each group; whereas, the area 'grid' model means there is a fixed number of columns of equal height (i.e. each column possess an equal number of rows). Despite the fact that all three multiplication models always produce the same product given the same problem (e.g. 3 x 5 = 15), all three multiplication models (i.e. repetitive addition, group and area) should be taught to students during core lessons.

However, since the teacher has been requiring students to master their multiples during the addition and subtraction math fact phase, students 'arrive' at their multiplication math facts with background knowledge of the operation's physical and conceptual meaning.

The vast majority of students quickly master their multiplication facts using the stair-stepped Formative Loop approach. As students take the mixed assessment, note the specific multiplication facts that they do not readily know. Again, create flash cards on only those specific multiplication math facts using 3 x 5 inch index cards.

Number Line Model: Multiplication of two (2) – 1 digit numbers



The product of 3 x 5 is a total of 5 equal jumps of 3 spaces each producing a product of 15.

Figure 5: Multiplication Via Repetitive Adding – Number Line Model

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Section 8

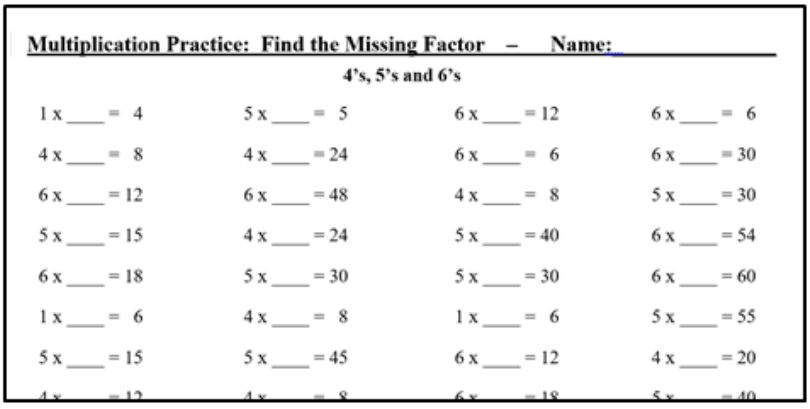
DIVISION FACTS – Classroom Specifics and Pedagogical Understanding

Early in the division phase as students begin the Formative Loop stepped-up sequence, they should be given quick 5 practice sheets of 'Finding the Missing Factor' practice sheets. Since students have mastered their multiplication facts, this focused practice assists but does not guarantee division math fact automaticity. Many teachers frequently preach to their students that, "division is just the opposite of multiplication." Consequently, division facts should be easy for them. However, for most students, it is not – since division requires a different skill set. The student is attempting to determine the missing 'factor' when the product and the other factor is provided; however, unlike multiplication, there is no commutative property for division. Analogously speaking, the student is trying to compute either the divisor or the quotient for division.

"Finding the Missing Factor" skill practice sheets prepare students for division, directly! There are three unique practice sheets for missing factor ranges [i.e. (0 to 3), (4 to 6) and (7 to 9)] available in the Formative Loop Resource Library. As with multiples and Making 10, these practice sheets for the 'Missing Factor' should be accurately completed in 5 minutes.

Once all three (3) 'Find the Missing Factor' skill practice sheets are completed, the vast majority of students master division facts immediately after completing the Formative Loop stair-stepped (i.e. 1's, 2's, 3's, etc.) sequencing. One (1) of the three (3) 'Find the Missing Factor' practice sheets is displayed in Figure 6 below.

Again, for any specific division facts the student struggles and has not specifically memorized, the teacher can create the customized flash cards using index cards for accentuated study.



Multiplication Practice: Find the Missing Factor – Name: _____

4's, 5's and 6's

1 x ___ = 4	5 x ___ = 5	6 x ___ = 12	6 x ___ = 6
4 x ___ = 8	4 x ___ = 24	6 x ___ = 6	6 x ___ = 30
6 x ___ = 12	6 x ___ = 48	4 x ___ = 8	5 x ___ = 30
5 x ___ = 15	4 x ___ = 24	5 x ___ = 40	6 x ___ = 54
6 x ___ = 18	5 x ___ = 30	5 x ___ = 30	6 x ___ = 60
1 x ___ = 6	4 x ___ = 8	1 x ___ = 6	5 x ___ = 55
5 x ___ = 15	5 x ___ = 45	6 x ___ = 12	4 x ___ = 20
4 x ___ = 12	4 x ___ = 8	6 x ___ = 18	5 x ___ = 40

Figure 6: Find 'The Missing Factor' Student Practice Sheet

Final Note: If division is being introduced for the first time or the student does not understand the connection between the mathematics and physical division model, then physical tactile and pictorial models need to be presented during the daily core lesson to ensure that students thoroughly understand the mathematics. The teacher should begin with small dividends and divisors with no remainders, and then transition to remainders. A pictorial model should be drawn with each problem until students demonstrate thorough physical understanding of the model.

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Section 9

Summary and Conclusion

Student math fact mastery has been the nemesis for so many students for far too long. When I began teaching in the early 1990's, I struggled, and I gave students one hundred (100) final mixed math fact problems repeatedly – some students memorized them easily and other students struggled the entire school year. The mixed practice was too overwhelming for many of them.

In the 1960's when I was an elementary student, I recollect my own elementary teacher's frustration with students that could not readily memorize their basic math facts. I imagine when my elementary teacher attended elementary school in the 1920's, her teacher could probably offer similar frustrations with their students and math fact memorization. Of course this cycle probably repeats back to the first arithmetic math lesson 3,000 years ago. But, it does not need to be that way. With a computer's ability to track, record data and efficiently control the sheet production, teachers can concentrate their classroom lessons on a highly effective pedagogy and sequencing so almost all students can viably learn their math facts to automaticity.

Of course, the last 3 to 5 children in each class will take some pressing and effort. But, it has been my professional experience in both low and high socioeconomic school settings, that a structured plan as described in this white paper can make this process much easier. As a matter of fact, in a large urban Title 1 school with ninety percent plus of students classified as economically disadvantaged, between 98% and 100% of Graham Elementary third (3^{rd}) grade students mastered all four math operations and multiples by year's end – for ten (10) consecutive years. The fourth and fifth grade math facts completion rate was at nearly 100% every school regardless of our 20 percent mobility rate since that small number of students is easily identified and academically targeted.

The mathematics success of Graham students on standardized State assessments nearly equaled our math fact student completion rates. The Formative Loop Numeracy Program in conjunction with the classroom pedagogy described in this paper produced not only heightened mathematics' performance, but greatly lowered teacher frustration and yearly turnover. The primary grade (i.e. first and second grades) Formative Loop implementation with similar accountability as our intermediate grades was an additional factor that directly affected our school's academic performance.

In as quick as one school year, mathematics student achievement can be accelerated to remarkable heights with effective classroom management, a structured numeracy program, core lessons that are conceptually designed with a focus on skill mastery and a grade level rigorous problem solving resource. In order to make large academic mathematics gains, there must be a focus on a daily numeracy program. The program must be consistently operated every school day – five (5) days a week with a focus on student performance. Time and attention to student math fact mastery is an essential part of the numeracy process since general process math skills such as even and odd numbers, place value, rounding, etc. are much easier for the students to master.

These pedagogical methods have resulted in unparalleled elementary Title 1 school academic success. The methodology outlined in this white paper has been successfully replicated by other urban and rural Title 1 schools with equaled student success. It is a simple system that any teacher or school faculty can emulate regardless of classroom experience. Replication of academic results only requires daily implementation consistency, similar classroom pedagogy described in the paper, and educators positively motivating, recognizing and celebrating their students' successes.

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NEXT STEPS – Campus Administration and Classroom Teachers

As a retired urban Title 1 principal, I recommend classroom teachers read this white paper, and they evaluate, reflect and discuss the following questions with the campus administration in either a weekly faculty meeting or a weekly grade level meeting. After finalizing agreements, the campus administration should follow-up to ensure instructional accountability in their campus classrooms.

1.) What is the quality level of the classroom Formative Loop numeracy implementation with regard to distribution of the **daily** five (5) minute assessment, monitoring and grading procedures?

2.) Is the numeracy program completed **daily** in the classroom with respect to value and importance?

3.) What is the quality of student interventions when students do not demonstrate mastery of a **daily** assessment?

4.) If students are not successful on a **daily** Formative Loop numeracy assessment on a skill that was previously covered in a core lesson, what does that indicate about core lesson instruction?

5.) Are teachers implementing 'spaced repetition' – mini lessons on multiples, making 10, and finding the missing factor methodology to prepare students for math fact success?

6.) Are teachers pressing all students for math fact success with homework, recognition and incentives?

7.) Are all classrooms on a grade level moving at approximately the same pace through the FL Numeracy Program? If not, what is the reason(s) for the performance and progress differences?

8.) Is the campus using the FL double-streaming sequencing (math facts and math processing skills) in 3rd grade to ensure high student math skill ability by the end of the school year?

9.) Are newly enrolled and academically struggling 4th, 5th and 6th grade students placed in a separate math fact sequencing for academic 'catch-up?'

Students cannot problem solve efficiently and effectively without fundamental arithmetic math skill mastery.

The elementary grades – including sixth (6th) grade must be deliberate and intentionally taught to ensure students soundly possess the rudimentary math skills. If not, as students progress in late middle school and high school dependent math subjects of algebra, trigonometry, geometry and calculus, they will not competently and successfully engage in those levels of mathematics.

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