Subtracting Whole Numbers Correctly – Every Time

Multi-Digit Whole Numbers
With and Without Regrouping

A simple and general subtraction technique
to eliminate calculation mistakes

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Executive Summary

After nearly 30 years of working in the public education system as a classroom teacher, administrator and consultant, it amazes me that I continue to see students subtract multi-digit whole numbers (and decimals) incorrectly. It also is bewildering that students complete the subtraction process correctly, and then, are not able to orally communicate the physical nature of the arithmetic operation of regrouping.

As a civil engineer that entered public education in my thirties, I feel compelled to write out a procedure with explanations and examples so that the interested classroom educator or administrator can employ it to rectify the arithmetic situation with their students. **As with all types of processes and procedures, the most effective one will always be a GENERAL method that encapsulates all situations.** For instance, it is immaterial if regrouping of a digit in the minuend is required or not, the process should be the same. Moreover, if a zero or a non-zero digit appears in the minuend, the subtraction process should be unchanged. The power of a general methodology always triumphs over a series of special case procedures since students only need to remember one method. The case of the general methodology is especially valuable since the mass of arithmetic mathematics is a Base 10 process. In short, a subtraction process must work every time for every set of whole numbers – regardless of their magnitudes.

Outside of basic place value understanding, the only prerequisite or dependent skill set is addition and subtraction fact mastery. All students can master their math facts if an effective daily numeracy program is pressed like Formative Loop. However, the school's administration must recognize that this skill set is a priority of importance and establish that skill mastery is a priority.

The salient points of this subtraction process are straightforward. The teacher **must** use specific vocabulary using the pedagogical technique, so ALL students understand the physical regrouping nature of mathematics as shown in the example problems. The benefits of the pedagogical technique are listed below.

1.) Vocabulary understanding: Minuend, Subtrahend, Difference, Addends, Sum and Fact Family.

2.) Subtraction of any two whole numbers – regardless of magnitude – always produces a difference between those two whole numbers. Precisely, the **difference** is the total number of equal spaces between the two whole numbers. Example: The one-digit subtraction process of: 8 – 5 = 3; means there are 3 equal spaces (i.e., the difference) between the two whole numbers – 8 and 5 on a number line. Furthermore, subtracting 31 from 98 or 98 – 31 yields a **difference** of 67. Therefore, there are 67 equal spaces between 31 and 98. Note: This understanding of difference in equal spaces between whole numbers/integers is key for student understanding of addition and subtraction of positive and negative integers in sixth grade.

3.) In a Base 10 system, there are 10 ones in 1 ten, and there are 10 tens in 1 hundred. Continuing, there are 10 hundreds in 1 thousand, etc., etc. Importantly, students must readily know equivalencies to possess a conceptual and physical understanding of the arithmetic/subtraction mechanics. The classroom teacher must verbally communicate these equivalent amounts during the direct teach and guided practice portions of the core lesson until students completely grasp these Base 10 equivalencies. These verbal communiques are provided in the enclosed examples.

4.) Addition is the opposite process of subtraction and vice versa. Hence, a simple addition equation can also represent a subtraction equation ~ specifically called a **fact family**. For instance, the fact family of 6, 2 and 4 is: 6 – 2 = 4 ; 6 – 4 = 2 ; 2 + 4 = 6 ; 4 + 2 = 6. These four interrelated arithmetic equations are called a fact family. Moreover, regardless of the size of multi-digit whole numbers of addition and subtraction, the same four arithmetic relationships are always valid. Example: There are three more arithmetic equations for 26 + 178 = 204 ~ two subtraction equations (i.e., 204 – 178 = 26 ; 204 – 26 = 178) and one more addition equation with the addends switched (e.g., 178 + 26 = 204).

5.) If students are taught to check their subtraction by adding up their calculated difference and subtrahend, then three important aspects of arithmetic mathematics are clearly understood. First, that addition and subtraction are relationally the same thing – a fact family. Second, that their subtraction work is checked each time for accuracy. Lastly, when subtracting whole numbers, a student visually and conceptually grasps the arithmetic process of regrouping a number with Base 10 place value equivalencies. Finally, during the checking process, students are reinforcing their addition skills as well.

6.) In recent years, standardized assessment writers provide potential answers on the multiple-choice test so that every possible mistake a student may make during regrouping/subtraction is given as a possible answer choice; thus, deceiving the student that they worked the subtraction equation mathematically correct – when in fact, they did not. Or a standardized assessment requires the student to write and submit their answer – an open-ended answer format. The student must confidently know that their subtraction work is correct every single time. In effect, the described subtraction process delivers that specific outcome every time. Additionally, the methodology includes a linear number line model that affords a visualization of the process. This linear model is also appearing in standardized testing to ensure that students clearly understand the concept and not only the physical mechanics of the subtraction process.

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The reason phonics is an important pedagogical literacy method is because it is a GENERAL method of decoding. Any method that utilizes a general process over a specific one will always govern in pragmatic use. Thus, children or adults only need to learn and remember one method – not a series of methods for specific cases. Subtracting whole numbers exemplifies this process – since there can be three common subtraction cases when the minuend (i.e., the top number) requires 1.) no regrouping, 2.) that regrouping is necessary, or 3.) regrouping when the minuend contains zero(es). Good pedagogy necessitates that students employ only one method in all three cases with the means to self-check the accuracy of their work. Finally, there is only one prerequisite skill in the subtraction process outside of basic place value understanding – addition and subtraction math facts. Math fact mastery is an **essential** skill to learn in arithmetic mathematics. **All students** can master both math fact and math processing skills using a daily numeracy program like Formative Loop, but the program must be **pressed** daily by both the campus administration and classroom teachers. Formative Loop is a numeracy program that presses **each** student individually using a blended writing component and digital tracking process to ingrain the desired skill.

There are three (3) examples below that expatiate subtraction with regrouping. Each student should be held accountable to the method until mastered. The teacher's (**T**) verbiage is also key – to ensure that students (**Sts**) are questioned for understanding during guided practice of the core lesson. These verbal interactions are written below or next to each step in the process. Note math vocabulary: Minuend, Subtrahend, Difference, Addends and Sum are consistently used to describe each whole number during the process.

EXAMPLE 1:

Initial Calculation

$$\begin{array}{r} 319 \\ -\ 187 \\ \hline \end{array}$$

Step 1

$$\begin{array}{r} 319 \\ -\ 187 \\ \hline 2 \end{array}$$

***T:** Can we take 7 cookies from 9 cookies?*
***Sts:** Yes.*

Step 2

$$\begin{array}{r} {\scriptstyle 2\ 10}\ \ \\ \not{3}19 \\ -\ 187 \\ \hline 2 \end{array}$$

***T:** Can we take 80 cookies from 10 cookies? 1 ten = 10 cookies. (Note: Yes, but not in Elem. School.)*
***Sts:** No. Must Regroup a 100 to 10 tens.*

Step 3

$$\begin{array}{r} {\scriptstyle 11}\ \ \\ {\scriptstyle 2\ \not{10}}\ \ \\ \not{3}\not{1}9 \\ -\ 187 \\ \hline 2 \end{array}$$

***T:** We have added 10 tens to our 1 ten. So, how many tens do we have now in the tens place?*
***Sts:** 11 tens - total. (Cross-out 10 tens and 1 ten – replaced with 11 tens.)*

Step 4

$$\begin{array}{r} {\scriptstyle 11}\ \ \\ {\scriptstyle 2\ \not{10}}\ \ \\ \not{3}\not{1}9 \\ -\ 187 \\ \hline 132 \end{array}$$

***T:** Can we take 8 tens from 11 tens?*
***Sts:** Yes.*
***T:** Can we take 1 hundred from 2 hundreds?*
***Sts:** Yes*
***T:** Subtract.*

Step 5

$$\begin{array}{r} 319 \\ -\ 187 \\ \hline 132 \end{array} \uparrow$$

***T:** Check your **Difference** by adding up using the **Subtrahend**.*

Step 6

$$\begin{array}{r} 132 \\ +\ 187 \\ \hline 319 \end{array} \checkmark$$

***T:** Our **Sum** matches our **Minuend.** Our Subtraction is correct!*

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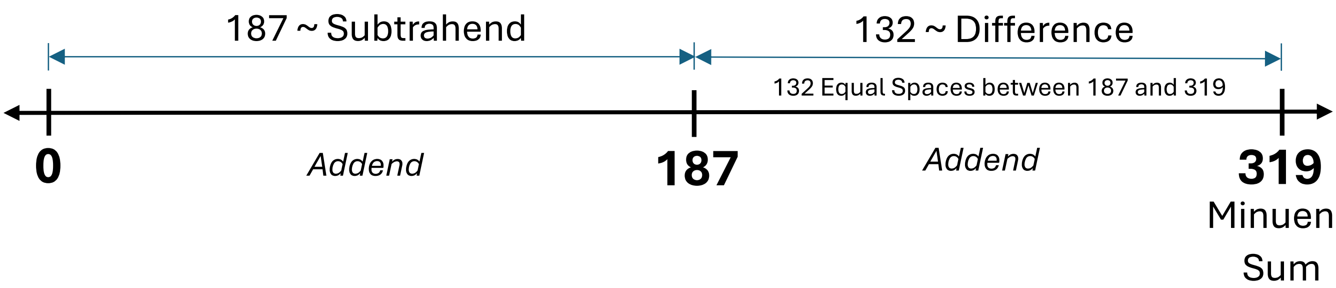
It is also helpful to provide students with a visual of the subtraction process using a whole number line for two reasons. First, it enhances their understanding both mechanically and visually/conceptually of the solution. Second, standardized testing is requiring students to demonstrate numeric understanding via number line models. Quick examples with a whole number line, and students rapidly ingrain the physical meaning of the process both numerically and visually/conceptually.



EXAMPLE 2:

Initial Calculation | **Step 1** | **Step 2** | **Step 3**

Step 1

T: *Can we take 8 cookies from 7 cookies?*
Sts: *No. Must Regroup.*
T: *Can we take 90 cookies from 80 cookies?*
Sts: *No. Must Regroup a 100 into 10 tens.*

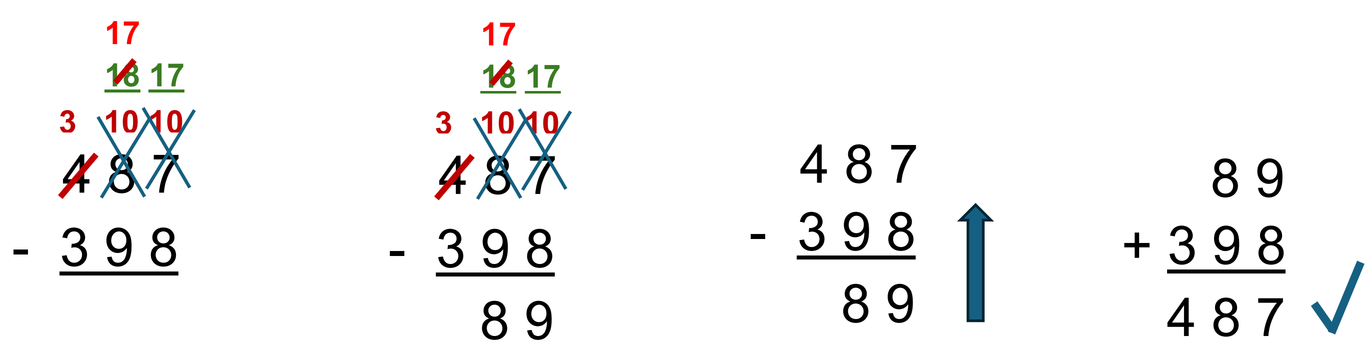
Step 2

T: *We have added 10 tens to our 8 tens. So, how many tens do we have **now** in the tens place?*
Sts: *18 tens - total. (Cross-out 10 tens and 8 tens – replaced with 18 tens.)*

Step 3

T: *What do we regroup next?*
Sts: *1 ten equals 10 ones. Regroup one of our 18 tens (Cross-out 7 ones and 10 ones – replaced with 17 ones.)*

Step 4 | **Step 5** | **Step 6** | **Step 7**



Step 4

T: *Are we ready to subtract?*
Sts: *Yes. We can take 8 ones from 17 ones. We can also subtract 9 tens from 17 tens.*

Step 5

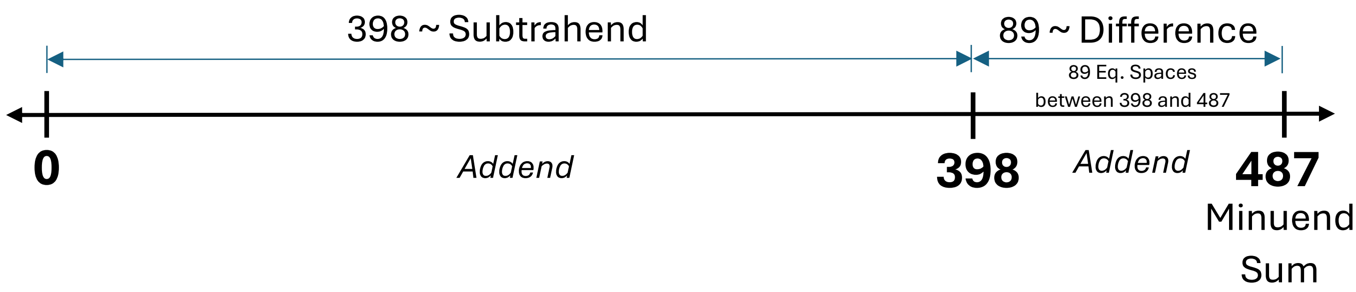
T: *Subtract.*

Step 6

T: *Check your **Difference** by adding up using the **Subtrahend**.*

Step 7

Sts: *Our **Sum** matches our **Minuend.** Our Subtraction is correct!*



EXAMPLE 3: **(Subtracting Across a Zero)**

Initial Calculation

602
\- 247

Step 1

5 10
~~6~~ 0 2
\- 247

T: Can we take 7 cookies from 2 cookies?
Sts: No. Must Regroup.
T: Can we take 40 cookies from 0 cookies?
Sts: No. Must Regroup a 100 into 10 tens.

Step 2

10
5 ~~10~~
~~6~~ ~~0~~ 2
\- 247

*T: We have added 10 tens to our 0 tens. So, how many tens do we have **now** in the tens place?*
Sts: 10 tens - total. (Cross-out 10 tens and 0 tens – replaced with 10 tens.)

Step 3

9
~~10~~ 12
5 ~~10~~ ~~10~~
~~6~~ ~~0~~ ~~2~~
\- 247

T: What do we regroup next?
Sts: 1 ten equals 10 ones. Regroup one of our 10 tens (Cross-out 2 ones and 10 ones – replaced with 12 ones.)

Step 4

9
~~10~~ 12
5 ~~10~~ ~~10~~
~~6~~ ~~0~~ ~~2~~
\- 247

T: Are we ready to subtract?
Sts: Yes. We can take 7 ones from 12 ones. We can also subtract 4 tens from 9 tens.

Step 5

9
~~10~~ 12
5 ~~10~~ ~~10~~
~~6~~ ~~0~~ ~~2~~
\- 247
355

T: Subtract.

Step 6

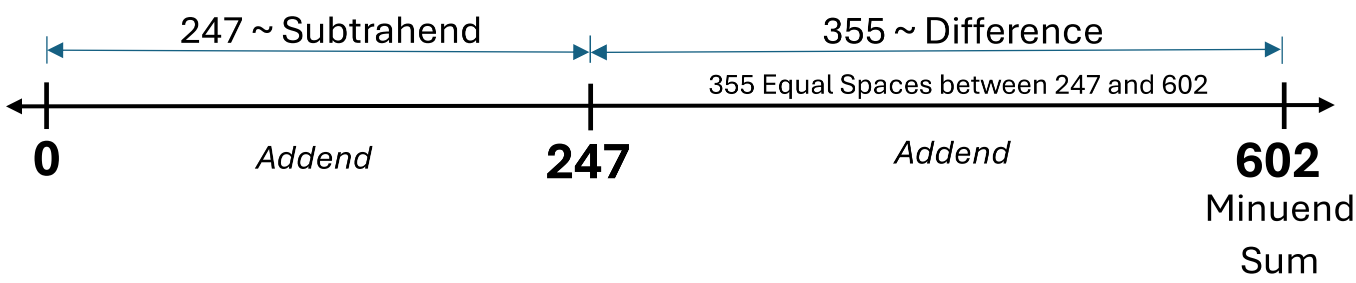
602
\- 247 ↑
355

*T: Check your **Difference** by adding up using the **Subtrahend**.*

Step 7

355
\+ 247
602 ✓

*Sts: Our **Sum** matches our **Minuend.** Our Subtraction is correct!*



Concluding Comments

The subtraction method shown above works in all possible subtraction cases – across zeroes, regrouping or not regrouping digits in the minuend. It is a general method which is a powerful tool in all mathematical processes. It additionally demonstrates the regrouping process conceptually, so students understand the means that regrouping is mathematically valid in a Base 10 system. The advantage of checking one's work by adding up the difference and subtrahend to find a sum equivalent to the minuend indicates to the student that their subtraction work was correctly completed. It also stresses the concept of fact family regardless of the size of the whole numbers – addition and subtraction are opposite arithmetic operations.

Finally, it is highly recommended that students draw a quick whole number line since it clearly demonstrates that they understand the physical elements of their arithmetic. Showing one's result on a number line is also a staple on many standardized tests; consequently, students are simultaneously prepared for that expectation of grade level rigor.

Math Vocabulary Review:

Subtraction:

7 9 2	← **Minuend**
- 1 4 5	← **Subtrahend**
6 4 7	← **Difference**

Addition:

6 4 7	← **Addend**
+ 1 4 5	← **Addend**
7 9 2	← **Sum**

Fact Family:

7 9 2
- 1 4 5
6 4 7

Four arithmetic equations:

792 – 145 = 647 ; 792 – 647 = 145

647 + 145 = 792 ; 145 + 647 = 792